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INVESTIGATION TOWARD PROVIDING THE BEST TECHNICAL APPROACH TO ATTAINING AND MAINTAINING RELIABILITY IN PROPELLER CONTROLS

BATTELLE MEMORIAL INSTITUTE
Columbus, Ohio

Propulsion Laboratory
Contract No. AF 33(616)-3344
WADC Project No. 3307

FEBRUARY 28, 1959

PROPULSION LABORATORY
WRIGHT AIR DEVELOPMENT CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE

Wright-Patterson Air Force Base, Ohio

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NOTICES

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FOREWORD

This report was prepared by Battelle Memorial Institute, of Columbus, Ohio, under USAF Contract No. AF 33(616)-3344, initiated under WADC Project No. 3307, Tasks Nos. 33064 and 33097, administered by the Propulsion Laboratory.

This report includes work conducted from January, 1956, to January, 1959, with emphasis on the final phase of study from June, 1957, to January, 1959.

Battelle Memorial Institute personnel contributing to this work:

Operations Research	David E. Debeau Donald L. Farrar William Drozda
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Battelle Memorial Institute

505 KING AVENUE COLUMBUS 1, OHIO

March 5, 1959

Commander
Wright Air Development Center
Wright-Patterson Air Force Base
Ohio

Attention WCLPBEL
Contract AF 33(616)-3344

Dear Sir:

Enclosed are 12 copies plus one reproducible copy of the final report, entitled "Investigation Toward Providing the Best Technical Approach to Attaining and Maintaining Reliability in Propeller Controls", prepared on Contract AF33(616)-3344. This final report is submitted in fulfillment of the requirements for the research program conducted by Battelle for the Propulsion Laboratory, Wright Air Development Center, during the period January, 1956, through January, 1959.

The results of the research program clearly show that an effective technical approach to reliability consists of not just one, but a number of, parallel, complementary actions on the part of the Laboratory and the contractors. An organized attack on the reliability problem can be successful; it involves the several steps of definition, testing, and evaluation suggested in this report.

Knowledge and understanding of the many facets of reliability are far from adequate. Progress has been slow because what has been learned is often applicable only to discrete cases under specific conditions. In the course of this research program, several promising areas have been revealed where further research could make substantial contributions to progress in reliability.

The role of redundancy in the success of past and present aircraft design is evident. However, its real potential and ultimate usefulness is only beginning to be realized. Questions of optimal configuration involving quantitative application of engineering cybernetics and systems techniques have hardly been explored at all.

Determination of accelerated test designs and evaluation criteria by means of applied statistical probability and model-theoretic analysis is a most promising but undeveloped area. As reliability requirements increase as a consequence of the increasing cost of failure, better ways must be developed to predict the expected frequency of events that occur so seldom that they may never be observed in tests or in service. Appendixes B and C in this report are first steps toward this goal.

The importance of environment in which uncontrolled forces inhibit system performance is undeniable. Exact knowledge of environment coupled with definitive statements of material properties should lead to theoretical laws of failure, and hence to the necessary generalization of such laws and useful applications. Until this can be accomplished, and as one method contributing to its accomplishment, the simulation of system performance by means of electronic computational aids is extremely useful in estimating system performance and reliability in an operational environment. The simulation presented in Appendix E is an example of the development of an initial simulation model for estimating the influence of reliability on operational capability. Thorough exploration of the variables selected for even this relatively simple model, not to mention the equal or greater number that had to be omitted, was not possible within the time and funds available. A skillfully designed simulation is most valuable as a research tool when it is used repeatedly to sift out the important variables, to identify significant parameter boundary values, and to point out continuously new directions for refinement and improvement of the simulation. Much more remains to be done in this respect.

We enjoyed this opportunity to work with Propulsion Laboratory and other Air Force personnel with interests pertinent to reliability in propeller controls. If there is any way we can be of further assistance, we will be happy to do so.

Sincerely,

E E Slowter

E. E. Slowter
Vice President

EES:pa
Enc. (12 plus 1 reproducible)

cc: Commanding Officer, Transportation Corps Army Aviation Coordinating Office WPAFB, Attention MCLATS-ED (15 copies)
Commanding Officer, Transportation Research and Engineering Command, Fort Eustis, Virginia (3 reports)
Chief, Dayton Air Procurement District (letter only)
Mr. Paul E. Weiser, WCKCN, WPAFB (letter only)

ABSTRACT

An effective reliability program must contain five essential activities: (1) definition of reliability terms and concepts, (2) specification of reliability goals and criteria, (3) measurement of reliability achievement, (4) comparison of measured reliability with specified goals and criteria, (5) and engineering action to attain and maintain reliability. This study describes specific steps in a technical approach to propeller-control reliability. Methodology of reliability, field experience, and a computer simulation developed during the research study are described in detail. Reliability is defined in terms of operational consequences of malfunction and failure. Numeric goals and criteria are identified in the context of design, test, and operation of propeller controls. Measurement is treated in terms of analyses and tests to be performed. A characteristic feature of the reliability "growth" process is the continuing need for comparisons of reliability estimates with numeric goals, followed by engineering action to attain and maintain reliability.

PUBLICATION REVIEW

This report has been reviewed and is approved

FOR THE COMMANDER:

Charles M. Michaels

CHARLES M. MICHAELS
Chief, Air Breathing Propulsion Division
Propulsion Laboratory

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INVESTIGATION TOWARD PROVIDING THE BEST TECHNICAL APPROACH TO ATTAINING AND MAINTAINING RELIABILITY IN PROPELLER CONTROLS

INTRODUCTION

Propeller controls for turboprop applications are designed to achieve automatic control of propeller operation in coordination with engine fuel control schedules. Propeller controls are vital elements in the propulsion system, and failure of these controls can have catastrophic consequences, in several ways. Failures resulting in propeller overspeed can lead to destructive vibration and disintegration of propeller or engine components. Failures resulting in loss of thrust during a critical maneuver such as landing or take-off or resulting in negative thrust can lead to loss of control of the aircraft or major structural damage.

The Propulsion Laboratory, Wright Air Development Center, is responsible for technical and operational development of propeller controls in the USAF. The Propulsion Laboratory recognized that operating characteristics of turboprop propulsion systems place more exacting requirements on the propeller and its control than its conventional predecessors. High thrust loading and wide range of blade-angle movement are required, as well as increased sensitivity and response rates, for the control function. The Propulsion Laboratory asked Battelle to conduct a study "To Determine the Best Technical Approach to Reliability of Turboprop Propeller Control Systems". This is the final report on this study.

This final report has been preceded by a series of four Battelle reports on particular aspects of reliability problems:

(1) "Formal Definitions of Reliability", May 20, 1957, dealing with the problem of defining reliability and providing a basis for relating reliability to operational experience (AD 132 094)

(2) "Significance of Estimates of Reliability Based on Truncated Sequential Tests of a Prototype With Repair of Failures", June 20, 1957, exploring the problem of endurance testing of prototype propeller controls and providing a method for estimating the assurance that the reliability of a control is greater than the required reliability when the required reliability, duration of tests, and a number of failures are known

Manuscript released by the authors 6 March 1959 for publication as a WADC Technical Report.

(3) "Reliability Analysis", July 22, 1957, discussing methods of estimating and predicting reliability in various states during development

(4) "Obtaining Reliability in Propeller Controls for Turboprop Applications", January 31, 1958, describing the basis for a continuing reliability program for propeller controls in the context of weapon-system development and USAF procedures for managing weapon-system-development programs.

The final report emphasizes the main points of the preceding studies and describes specific steps in a technical approach to propeller-control reliability. The principles and procedures for attaining reliability are in the main applicable both to propeller controls and to other functional components of weapons systems. Detailed results of the final phase of the study are presented in the Appendixes. The main objectives in this final phase are to gain understanding of the impact of reliability testing, operational environment, and criteria of success on equipment reliability in service and to examine possible approaches to these problems. The basic concepts of reliability are reviewed briefly in Appendix A. The problem of reliability testing and verification is explored in Appendix B. Appendix C presents a novel approach to accelerated testing as a problem of modeling. An analysis of field experience with turboprop systems is presented in Appendix D. Appendix E describes a digital-computer simulation of turboprop aircraft system operations under alternative conditions of operational demand and reliability.

PLANNED GROWTH OF A WEAPON SYSTEM

At the time an operational requirement for a weapon system is generated, there is a general notion of the reliability required if the system is to be operationally successful. Reliability finds quantitative expression at this early stage in the form of estimates of the force size required to perform the prescribed mission, allowing for availability, abort rates, noncombat losses, maintenance, and logistic support. These are gross estimates, based on experience with other systems of similar type operating in a wide variety of situations. These early estimates cannot be stated in terms of specific reliability levels. Specific reliability levels will be dependent upon particular design features of the system yet to be developed. Therefore, these estimates represent goals that subsequent development actions must achieve.

More specifically, the USAF operational goals in which reliability plays an important part include the following:

(1) To maximize the probability of success for the next mission

(2) To maximize availability for the next mission

(3) To minimize total risk during the operational life of the system

(4) To minimize total costs during the operational life of the system.

When an operational requirement has been established, the next step is to estimate the feasibility of designing, developing, and producing the weapon system to perform the intended function. This is usually accomplished by obtaining preliminary designs, first estimates of engineering feasibility and costs, and an evaluation of the problems standing in the way of successful development. In modern complex systems, reliability is one of the problems that may cast doubt on the potential success of a development program. Heretofore, reliability has seldom been evaluated until other performance requirements were attained. Experience shows that reliability must be considered from the beginning of development in order to assure that equipment with acceptable inherent reliability may be produced.

If it is decided to develop the weapon system, plans and programs must be established to guide and evaluate the development effort toward the desired goals. Advance preparation for planning the "growth" of a weapon system can be as difficult and problematical in many ways as the development work that is to follow. This is presently true in planning reliability "growth". The statement of requirements for subsystems and components should state the reliability goals as well as the performance goals to be achieved. Procedures for evaluating and testing the development items must be determined, and the basis for acceptance of the final products established. Before development can proceed, there is at present a need for more information regarding the technical aspects of design and the operational environment relevant to reliability.

A plan for achieving reliable propeller controls, the subject of this study, should be a part of the plan for developing the propulsion system, and, hence, the complete aircraft system.

PLANNED GROWTH OF RELIABILITY

"The objective of a reliability program is to achieve the highest degree of reliability consistent with design objectives, economic constraints, and the operational mission concerned. A practical program for reliability must recognize the facts of life about the environment in which it will operate. It should be consistent with existing USAF development policies and procedures. It must possess a logical sequence of development steps for 'growing' reliability. It must fit into present development programs as a part of the program requirements. Finally, it must provide a feedback of useful information for improving future reliability programs."

The above is quoted from a previous report[1] on this contract. It states some fundamental truths for reliability programming and identifies the achievement of reliability as a "growth" process. The reliability goal from the USAF viewpoint is the reliability ultimately attained in operations. Not only does this involve the functional integrity and inherent reliability of the system, but it must account for the effects of assigned task, operational policy, doctrine, maintenance, and the entire operational environment. Further, performance of the system, although essential, can pertain to one-time accomplishment, whereas reliability always implies repeated accomplishment over a period of time. To "grow" reliability for complex equipment, one must start from meager beginnings and design, test, and redesign until the probability of failure is low enough to provide for the required reliability in operations, in spite of the inevitable operational degradation.

There appears to be no single action that in itself can achieve reliability. Instead, the technical approach to reliability with the best chance of sucess consists of a number of actions pursued throughout propeller-control design, development, production, and operation:

(1) Centralize responsibility for the reliability program.

(2) Establish reliability requirements and plan approaches to reliability "growth" in advance.

(3) Perform, review, and evaluate reliability analyses.

(4) Conduct and evaluate reliability tests.

(5) Collect and disseminate reliability information.

(6) Monitor reliability "growth" throughout the life of the system.

For the weapon systems over which it has cognizance, WADC "is essentially the prime contractor for the actual research and development effort for a given program"[2]. The logical location for centralized

(1) Debeau, D. E., Farrar, D. L., et al., "Attaining Reliability in Propeller Controls for Turboprop Applications", Special Report, USAF Contract No. AF 33(616)-3344, Battelle Memorial Institute, January 15, 1958.

(2) ARDC Manual 80-4, dated 1 September 1956.

responsibility for propeller-control reliability is the Propulsion Laboratory. The Laboratory should provide definitive guidance for the development and operational use of propeller controls and similar control components in the form of reliability definitions, requirements, goals, environmental-design information, test and evaluation procedures and criteria, reliability-analysis procedures, component and system performance histories, and recommended inspection, maintenance, overhaul, and product-improvement methods. Each of these actions contributes to reliability growth during the evolution and life of the control. Each action requires continuous effort on current developments and in preparation for future developments.

The Laboratory must be prepared to state reliability requirements and actions to be taken to demonstrate that these requirements have been satisfied at the following points in the life cycle of a propeller control:

(1) Preliminary Design: The design study should indicate gross reliability requirements and the technical basis for evaluating reliability in subsequent laboratory and flight tests.

(2) Statement of Work: The statement of work should indicate preliminary estimates of propeller-control reliability requirements and preliminary test plans for laboratory evaluation of reliability that have been shown to be feasible.

(3) Reliability Analyses: Reliability analysis of initial designs should be based on the best available reliability data from WADC sources and industry experience.

(4) Component Evaluations: Component evaluations should be based on selected test conditions derived from previous findings of reliability analyses, analyses of the operational environment, and extensions of model-theoretic formulations for accelerated tests.

(5) Laboratory Tests: Qualification tests, type tests, and experimental flight clearance tests should be supplemented by selected accelerated tests for evaluation of reliability with respect to emergency and catastrophic consequences. These tests should be complemented by reliability analyses reflecting current design configuration.

(6) Production: Inherent reliability demonstrated by the manufacturer and WADC should be preserved in the production process by means of sequential-sampling quality-control methods that include short- and long-duration running of a sample of the production

controls. Consideration should be given to the development of an accelerated test to replace the very long endurance tests now given to a very small sample of production controls.

(7) Flight Tests: Flight tests (Categories I and II[1]) should be monitored to provide specific failure data as to cause, effect, frequency, and operating history to be used in revising reliability analyses and the selected conditions for laboratory and manufacturer production tests.

(8) Operations: Changes in operational reliability should be monitored to provide engineering data as well as reliability data for design changes and revision of reliability estimates.

In the following paragraphs, certain specific recommendations are made for Laboratory action toward attaining and maintaining propeller-control reliability in present and future weapon systems.

STATE GROSS OPERATIONAL RELIABILITY REQUIREMENTS FOR THE SYSTEM AND REFINE THESE VALUES AS DEVELOPMENT PROCEEDS THROUGH DESIGN, PRODUCTION, AND OPERATIONAL USE.

Analyses similar to those in Appendix E may be available initially from systems requirements studies preceding the development program. The gross values for operational reliability are derived at first from estimated flying loads (flying hours per unit time), organizational strengths, and other operating conditions in comparison with specific mission requirements that determine the allowable minimum availability rates and maximum abort rates, repair rates, and logistics. Propeller controls are but one component of the aircraft system contributing to the operation. As a vital component, experience shows that such controls should contribute less than 10 per cent of the total abort, repair, or logistic burden, the exact value being a matter of judgment. Initially, gross reliability values for categories of failure are estimated on the basis of an average of past operational experience for aircraft available, mission completion, and malfunctions per unit time. As experience is gained with the controls, more accurate and complete evaluations should be possible.

(1) AFR 80-14, 19 August 1958.

DEFINE RELIABILITY AND DESIRABLE MINIMUM VALUES FOR SPECIFIC RELIABILITY CATEGORIES (Appendix A): CATASTROPHIC RELIABILITY, EMERGENCY RELIABILITY, FIELD-MAINTENANCE RELIABILITY, OPERATING LIFE.

Definitions of reliability and categories of reliability based on "mean life" concepts are described in Appendix A. Numeric goals for each category of reliability may be stated, but estimates of the reliability of the control cannot be made until discrete design configurations have been identified. When each configuration has been defined, estimates of reliability should be compared with goals such as the following:

(1) Probability of propeller-control malfunctions with catastrophic consequences should be less than 1×10^{-7} per flying hour.

(2) Probability of propeller-control malfunctions with emergency consequences should be less than that value for which combined emergency malfunctions can produce catastrophic consequences more frequently than 1 in 10^7 flying hours (e.g., for a four-engine, turboprop aircraft with independent controls, probability of emergency malfunctions should be less than 3×10^{-3} per flying hour).

(3) Probability of propeller-control malfunctions resulting in field-maintenance requirements should be less than 0.01 per flying hour.

(4) Probability of propeller-control malfunction resulting in removal for overhaul should be less than 0.001 per flying hour. For this value, the fraction of the initial number of controls surviving to 1500 hours would be 0.22.

In the design stage, the confidence levels for the reliability estimates are likely to be quite low, i.e., below 0.50. This does not mean that reliability is low, but that knowledge required to narrow the confidence interval is lacking. As experience is gained through physical tests[1] and reliability analysis[2], engineering estimates of likely causes of failure and probability

(1) Kuhn, G. R., Debeau, D. E., and Swager, W. L., "Significance of Estimates of Reliability Based on Truncated Sequential Test of a Prototype With Repair of Failures", Special Report, USAF Contract No. 33(616)-3344, Battelle Memorial Institute, June 20, 1957.

(2) Debeau, D. E., Farrar, D. L., et al., op. cit.

of occurrence of malfunction will improve. With this additional information, estimates of reliability with higher confidence levels should be possible. Before production is initiated, every effort must be made to estimate the reliability values at least at the 0.90 confidence level.

CONTINUE DEVELOPMENT OF FUNDAMENTAL APPROACHES TO RELIABILITY, QUANTITATIVE CRITERIA, AND METHODS.

Development of a model-theoretic approach to reliability in Appendix C represents an enlightening departure from "mean life" concepts of failure, particularly for catastrophic reliability requirements. The primary concern in predicting catastrophic failures is with the "tail" of the probability density function, where early failures occur. This novel approach concludes that "mean-life" requirements are not appropriately applied to catastrophic reliability of highly reliable systems. For example, it is proposed that the catastrophic requirement take the following form: "Demonstrate with 90 per cent confidence that the probability of catastrophic failure of the system with 10^4 operating hours under normal use is less than 0.001."

PLAN TO PROVIDE TECHNICAL AND OPERATIONAL DATA FOR DESIGN, DEVELOPMENT, AND PRODUCTION PHASES OF THE PROGRAM.

The Laboratory should supplement the information presently made available to design, development, and production functions with as many data as possible pertaining to reliability experience on systems, components, and parts related to the field of interest. There is at present no adequate source of reliability-engineering data for mechanical, hydraulic, or pneumatic equipment as there is for electrical and electronic equipment. Military specifications in their present form are generally inadequate for reliability estimation when the application in question involves marginal performance or alternative environmental conditions. Because reliability estimates in the design and development phases depend upon engineering judgment to a great extent, there is real need for comparative information for this purpose.

OBTAIN VALID FUNCTIONAL DESCRIPTIONS AND RELIABILITY ANALYSES OF EACH CONTROL CONFIGURATION SUCH THAT SEQUENTIAL, INDEPENDENT, AND REDUNDANT FEATURES OF DESIGN ARE CLEAR, AND PRINCIPLES OF STATISTICAL PROBABILITY ARE PROPERLY APPLIED IN ESTIMATING THE PROBABILITY AND CONSQUENCES OF FAILURE.[1]

Reliability analyses based on valid functional descriptions of the propeller control[2] are the main feature in any reliability effort. Vague comparisons of complexity attempted in the past are not adequate. The analysis must consider the effects of failure, relation of parts and components to each other in a dynamic sense, and specific stress conditions involved. Detailed rules cannot and should not be stated at this time. Each problem is singular in some way, and engineering judgement and skill are required to perform the analysis for each configuration and each phase of development. Reliability analyses of production and operational equipment should have the benefit of data from tests and field experience to establish cause-and-effect relationships and firm estimates of reliability measures for observed failure events. Thus, reliability can become as demonstrable as other performance qualities. However, prompt feedback of test and field data is necessary in view of present accelerated procurement delivery schedules.

SUPPORT COMPONENT-EVALUATION TESTS BY MANUFACTURERS.

Evaluation of the components independent of the system is properly a consideration of the manufacturers. The Laboratory should support the manufacturers with collateral information on component test design and experience from previous developments and information relating the results of previous component tests to usage experience with these components.

(1) Baum, J. V., Kuhn, G., Debeau, D. E., and Farrar, D. L., "Reliability Analysis: A Part of an Investigation Toward Providing the Best Technical Approach to the Reliability of Propeller Controls for Turboprop Applications", Special Report, USAF Contract No. AF 33(616)-3344, Battelle Memorial Institute, July 22, 1957.

(2) Baum, J. V., Kuhn, G., Debeau, D. F., and Farrar, D. L., op. cit.

PLAN LABORATORY RELIABILITY TEST PROGRAMS IN ADVANCE TO SPECIFY THE METHODS OF EVALUATING THE RELIABILITY ASPECTS OF PROPELLER CONTROLS.

Prior to the design and development of a propeller control, the Laboratory should specify the testing program that will be used to evaluate the reliability aspects of the control. This information is essential guidance both to the designers and to the Laboratory's own project officers.

The tests conducted under the cognizance of the Laboratory include Qualification Tests, Type Tests, and Flight Clearance Tests. Although these tests are intended mainly to produce functional evaluations, some of them may also provide useful information for reliability evaluation if provision is made for collection and evaluation of such information in advance.

The tests run today are primarily nonaccelerated tests under idealized environmental conditions. Since the real operational environment cannot be simulated exactly, these tests provide an estimate of inherent reliability within the design conditions specified, rather than an estimate of operational reliability. The Laboratory should study operational environment on a continuous basis in order to provide an improved basis for test design. This continuous study involves evaluation of observed operational failures for interpretation as to causes and consequences and statistical patterns.

For each test, the Laboratory must decide the purpose and the information desired with respect to reliability. For instance, it may be decided that an experimental flight clearance test should provide a 50 per cent assurance that, under laboratory running conditions, the mean time to a malfunction with potentially emergency consequences should be greater than 300 hours. The Laboratory should then use a statement of requirement such as the one above to specify the number of hours of testing to be accomplished with no malfunction with potentially emergency consequences. Using the hypothetical example above and Figure B-2 in Appendix B, it may be estimated that at least 240 hours of running by the Laboratory would be required.

Propeller controls are vital elements in the aircraft system. The reliability requirement for such vital elements is, by definition, very high (i.e., probability of catastrophic consequences of malfunction less than 1×10^{-7} per flying hour and probability of emergency consequences of malfunction less than 3×10^{-3} per flying hour). Nonaccelerated laboratory tests cannot economically provide any reasonable assurance that these high reliability levels have been achieved.

Accelerated laboratory tests specifying selected sets of environmental conditions, abuse ratios, hours of testing, and sample size appear to be feasible in the model-theoretic sense illustrated in Appendix C. The

Laboratory should continue development of this approach as the only presently identified means of evaluating the reliability of highly reliable systems.

USE RELIABILITY EVALUATIONS, ANALYSES, AND TESTS TO DETERMINE THE NEED FOR ENGINEERING ACTIONS TO IMPROVE RELIABILITY.

The statistical evaluations that provide estimates of reliability do not necessarily indicate the actions to be taken in improving the design. The actions required are engineering actions, based on knowledge of the physical properties and stresses involved. The assurance with which reliability can be estimated is directly dependent upon the precision with which the stress/strength relationships in the system environment are known, as well as on the duration of testing. In early development phases, lack of precise knowledge about stress/strength relationships in the operational environment is responsible for the low assurance in reliability estimates.

Engineering actions that may be taken to improve reliability are of several types. Choice of action is a matter of engineering skill and judgment, with appropriate reliability estimates providing additional guidance.

Reliability can be improved by selection of materials that provide more favorable strength/stress relationships in specific cases. The dimensions and other critical characteristics of parts and components made from these materials can be altered to increase failure thresholds and time between failures. There is often a choice among alternative mechanisms for accomplishing the functions of force transmission or control. One criterion would be estimated reliability of the system for each alternative. Design configurations based on functional requirements are likewise a matter of choice, and reliability is one criterion. The use of redundancy to parallel or provide "fail-safe" operation of major assemblies as opposed to component redundancy is often an effective engineering design method. The actions taken to improve reliability, then, are engineering actions. The need for action is determined by the reliability evaluations, analyses, and tests.

ESTABLISH CLOSE LIAISON WITH OPERATIONAL COMMANDS PHASING IN NEW PROPELLER-CONTROL MODELS IN ORDER TO OBTAIN OPERATING RECORDS SUPPLEMENTING.

RELIABILITY DATA FROM PRODUCT-IMPROVEMENT PROGRAMS OR OTHER SOURCES.

Results of the field-data-collection program, presented in Appendix D, indicate the value of first-hand information on operating history. Such information can be obtained without interfering with operational requirements of the Commands. For operational reliability estimates, it is essential to know the effects of malfunctions, operating-time history of each control, and maintenance/logistic support environment for a period not less than the 50 per cent survival time estimated for the control. In this study, the DD Form 781-2 proved a useful source of information. This form has been superseded by AFTO 781-A.

USE COST COMPARISONS CAUTIOUSLY IN EVALUATING RELIABILITY FOR DEVELOPMENT PROGRAMS.

The costs of design and development that can appropriately be charged to reliability as opposed to other design requirements cannot be established with any reasonable accuracy at this time. Costs incurred in service as a result of reliability limitations or other operating problems are subject to wide latitude in interpretation. Both of these cost areas are basic inputs to a cost analysis of the relationship between development effort and utilization costs. Until there is substantial improvement in the available quantitative estimates of such costs, it does not seem advisable to base cost limitations for reliability development upon a comparison of this nature.

CONCLUSIONS

Planned growth of reliability through effective application of the reliability concepts and procedures described herein:

(1) Will create a better understanding of reliability problems in the design and development of propeller controls and other components of weapon systems

(2) Will afford the means for prompt, positive action toward attaining and maintaining reliability in weapon systems and their components.

APPENDIX A

CONCEPTS OF RELIABILITY

APPENDIX A

CONCEPTS OF RELIABILITY

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APPENDIX A

CONCEPTS OF RELIABILITY

Fundamental Notions

The fundamental notion of reliability or dependability has existed throughout history. Reliability has signified qualitative judgment expressed as some degree of confidence that a device or even a person would in the future perform some intended function. There was no quantitative measure associated with this judgment, only a subjective statement of confidence. Reliability was considered a desirable quality and, in many cases, particularly military applications, an essential one. Until recently, no serious attempt was made to state reliability in quantitative terms because it was known to be dependent upon the explicit conditions surrounding a particular situation and, thus, was highly variable. Present determination to measure reliability and to use it as a criterion for performance stems from the need to evaluate future situations in terms of two universally important factors, risk and cost. Risk involves the probability of failure and the attendant losses expected as a result of that failure and, hence, introduces the notions of statistical probability and expectancy. Cost is the economic measure of relative gain or loss on some arbitrary value scale.

Failure Rate

In the intuitive sense, reliability is associated with, if not in fact measured by, frequency of failure its inverse function, the interval between failures. It has been shown that complex equipment consisting of a relatively large number of parts exhibits certain characteristic patterns of failure over time. The actual quantitative values involved may vary, depending upon environment, but the form of the pattern remains essentially the same. Different equipments will show one or more of the different types of failure involved.

The patterns of failure observed for complex equipment are shown in Figures 1 and 2. The difference between them is simply that Figure 1 relates failures per unit time to the number of surviving equipments operating in that unit of time, whereas Figure 2 relates failures per unit time to the original or initial number of units, failures thus decreasing as the number of surviving units decrease. Otherwise, both express identical rates of failure and can be discussed simultaneously.

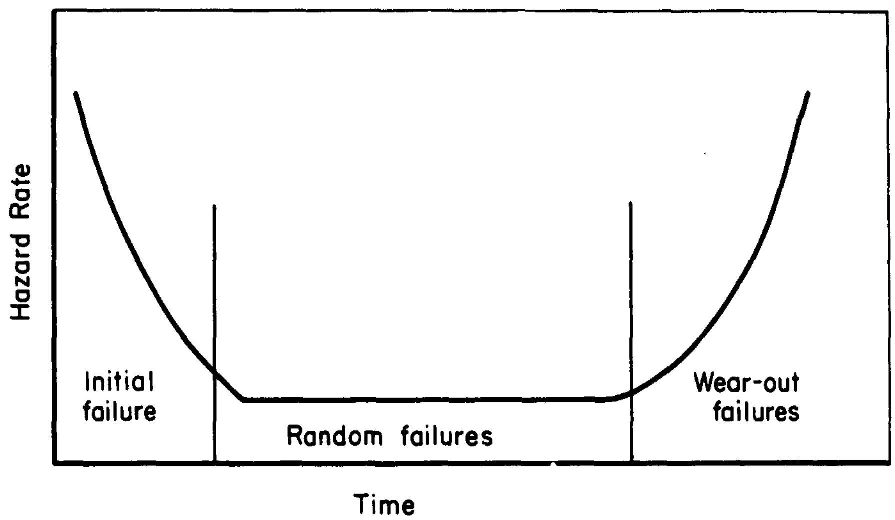

FIGURE 1. TYPICAL FAILURE-RATE (HAZARD-RATE) CURVE BASED ON NUMBER OF SURVIVING UNITS

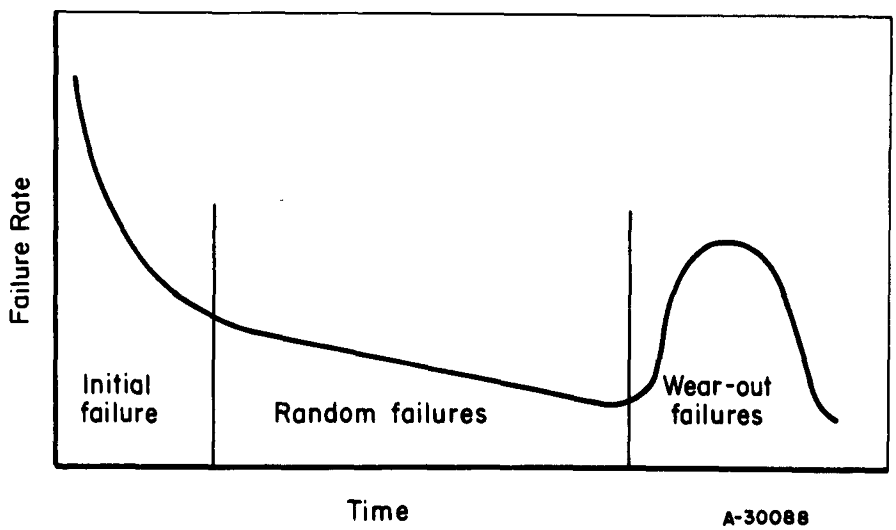

FIGURE 2. TYPICAL FAILURE-RATE CURVE BASED ON INITIAL NUMBER OF UNITS

The initial failure rate or "infant mortality" failures result from manufacturing defects, damage in shipping or handling, or errors in installation. To some degree, all equipment will always be subject to such failures. The incidence of manufacturing defects and marginal parts can be reduced by improved manufacturing or quality-control procedures or "green-running". Very often these parts are not recognizable as defective until they fail.

The random-failure or "normal" operating period is that period during which failure rate is essentially constant and failures can occur with equal probability at any time. Foreign-object damage is one example of the uncontrollable failures that can occur during this period. The duration of the random-failure period depends upon the aging process or the occurrence of wear-out and the operational environment, represented by duty cycles, severity of use, maintenance, etc.

The last period is the "wear-out" period, during which the failure rate rapidly increases as more of the equipments reach their critical age and begin to deteriorate. This period of failure can be avoided entirely by replacement of parts before they reach critical age.

Categories of Failure

The difficulty in quantifying reliability stems in part from the more basic problem of defining failure in quantitative terms. In the intuitive sense, a failure is an unsuccessful attempt to perform some action. Again, failure is a subjective judgment in that the degree of unsuccessfulness is not specified. Since reliability is contingent upon the nonoccurrence of failure, the definition of failure is fundamental in the concept of reliability.

Failure may be defined subjectively as the inability of equipment to perform its required function (Reference 37), or operation outside of design tolerances, or failure to meet specified operational requirements. These and other current definitions of failure are posed in terms of the consequences of failure, rather than the essence of failure itself. Comparatively little is known about why materials fail, why environmental influences such as temperature, humidity, vibration, moisture, or radiation cause deterioration of material strength. The cause-and-effect relations now employed to express failure characteristics avoid the unknown essence of failure. This, in part, is responsible for the present use of statistical probability relations, where, if they were known, physical laws of failure would be more accurate and revealing.

Since the weight of evidence is presently in the consequences of failure, the categories of failure selected should reflect this state of affairs. Some categorical schemes suggested by manufacturers and users of propeller controls are shown in Table 1. Failure is defined in terms of its consequences, rather than its causes, and is meaningful from an operational point of view in terms of the fraction of failures that cause mission failure.

Another scheme classifying failure in accordance with its relation to intended function gives a clearer picture of the nature of failure causes and effects on mission success. In Table 2, two classes of stress are distinguished, extrinsic stresses and intrinsic stresses. Extrinsic stresses are those originating outside the immediate environment of the part. Intrinsic stresses are those functional stresses inherent in the functional design and which the parts were designed to withstand. Three categories of outcome of a failure event are indicated: the effect on the part, the consequence to the system, and the result of the operational mission. From an operational viewpoint, only the result of the mission is of immediate importance. Maintenance as a consequence of system malfunction is a second-order effect.

Hence, a distinction between failure and malfunction must be drawn in the case of propellers and their controls and for many other equipments.

The effect of each possible failure on the component should be identified. This is the first step in identifying component failures with reliability categories. A component failure may cause a component to be completely inoperative, to operate in an erratic fashion, or to operate entirely incorrectly. A broken push rod or a vacuum tube with a burnt-out filament would be examples of inoperative components. An electric relay with an eroded contact or a weakened spring such that it occasionally operated when no operation was desired would be examples of erratic operation. Examples of incorrect operation might be a drift in the value of a condenser in a timing circuit, causing a change in the time constant of the circuit, a slipping clutch, or a leaking hydraulic valve.

The effect of each possible failure on the operation of the propeller control should then be identified. As an example, assume that a switch that actuates the emergency feathering of a propeller fails by closing, perhaps as a result of vibration, when closing was not desired by the pilot. The effect of this failure upon the component would be an undesired operation. The effect of this failure on the propeller control would be an undesired control operation. Drift in the value of a condenser in a timing circuit could lead to a lag in the response of the control. A slipping clutch could lead to an incorrect blade-angle setting by the control.

Finally, the effect of a component failure on the airplane in terms of categories of reliability previously set forth must be identified. Referring again to the example of the failure in the emergency-feathering

TABLE 1. SCHEMES FOR CATEGORIZING FAILURES

A	B	C
Critical Defect – a defect that experience indicates could result in hazardous or unsafe conditions or prevent performance of the intended function	I-Serious – a defect that can cause failure or malfunction prior to overhaul and can be detected only by disassembly	Primary Failures – those failures requiring immediate corrective action to retain control of the engine or aircraft
Major Defect – a defect other than critical that could result in failure or could materially reduce performance of the intended function	II – Serious – a defect that can cause failure or malfunction prior to overhaul and can be detected without disassembly	Contributory Failures – those failures not immediately apparent and that may not affect flight characteristics except when followed by a primary failure
Minor Defect – all others	III – Minor – a defect that cannot cause failure or malfunction prior to overhaul	Noncontributory Failures – those failures that do not affect safety of flight
	IV – Other – a defect caused by disassembly	

TABLE 2. EVALUATION OF A FAILURE EVENT

Stresses Leading to Failure		Outcome of Failure Event		
Extrinsic Stresses (External to the Part)	Intrinsic Stresses (Inherent in the Part)	Effect (On the Part)	Consequence (To the System)	Result (Of the Operational Mission)
Physical environmental conditions: temperature, shock, vibration, sand and dust, moisture, radiation	Design error	Rapid wear-out	Stoppage	Destruction
	Quality-control error	Rupture	Partial stoppage	Abort
		Drift	Deterioration	Degraded performance
				No effect
Human influences: operator error, maintenance error				
Design error				

switch, this failure would cause feathering of the propeller and loss of power from the engine. For a multiengined airplane, this failure could be no more serious than a requirement for a field-maintenance action.

There is not a sharp "go-no-go" or failure effect for the vast majority of malfunctions. Further, reliability must be evaluated in terms of its operational effect and on the basis of the intended function and environment of the equipment. Interdependency of part, component, and system malfunction and failure is a distinct characteristic of the system design. The analysis of reliability discussed in another part of this report requires detailed knowledge of these relationships and the effects of the particular environmental conditions anticipated.

Formal Definitions of Reliability

From a technical point of view, the reliability of a propeller control is the probability that such a control will perform for some unit of time without malfunction as a function of assigned task, environment, and previous history. The measure or yardstick for reliability is a probability that a given event will not occur. Even for a single well-defined system, such as a propeller-control system, there are many possible definitions of the terms "perform", "malfunction", and "assigned task". Thus, stemming from this single technical definition, there are a large number of specific definitions of reliability.

Catastrophic Reliability

The "catastrophic" definition of reliability is already recognized at all levels in the USAF and among the contractors as the single most important criterion of reliability. No matter how it is worded or referred to, it appears in the thinking and actions at every stage of development and operation. There is no question but that this emphasis is correct.

> By definition, "catastrophic" reliability is the probability that a given propeller control will perform for a period of time without a malfunction resulting in catastrophic consequences for the aircraft.

Generally, it is easier to state this in terms of the probability that a catastrophic malfunction will occur. The smaller this number, the more reliable the system.

Emergency Reliability

The next most important definition of reliability would appear to be the "emergency" criterion.

> By definition, "emergency" reliability is the probability that a propeller control will perform for a period of time without a malfunction requiring manual feathering of a propeller by the pilot or other equivalent emergency action.

Typical of what is visualized as an emergency condition equivalent to a manual feathering preventing effective use of a turboprop power plant would be a condition of complete internal control failure, resulting in actuation of the propeller pitch lock and loss of manual feathering capability.

Maintenance Reliability

Two maintenance concepts of reliability appear to be important, the "depot maintenance" concept and the "field maintenance" concept.

> The fraction of operating controls that reach the mandatory overhaul age without premature removal is a measure of the reliability of the control in the context of the "depot maintenance" concept of reliability.

The number of operating hours between major overhauls at the depot may be thought of as the "operating life" of the control. After completing a predetermined number of operating hours, systems such as the propeller control are removed from the aircraft for depot overhaul. This predetermined number of hours is termed the mandatory overhaul period. If a control exhibits a malfunction that cannot be repaired by line or field maintenance before reaching the mandatory overhaul period, it is removed from the airplane and sent to the proper depot for overhaul. This action is termed a premature or irregular removal.

Alternative statements of this concept of reliability include (a) fraction of systems surviving a specified number of flying hours, (b) number of flying hours a specified fraction of systems are expected to survive, and (c) the fraction of systems surviving to the mandatory overhaul age.

> A useful operating definition of reliability in terms of the "field maintenance" concept is the number of malfunctions that occur per 1000 flying hours that can be corrected by field maintenance.

The smaller this number, the greater the reliability of the system.

Measure and Reliability

The numbers used to describe the level of reliability attained or desired are not numbers that can be arrived at by direct measurement. Rather, these numbers represent a prediction of what is expected to happen or not to happen in future use of the equipment and, hence, must be developed by some logical process from numbers that can be measured directly.

As in every statistical study, the assignment of numerical values to reliability and related concepts depends on the assumption that history repeats itself. The life history is recorded. The number of items that fail in each time interval are then tabulated or plotted as a histogram. In some cases, it is possible to represent the empirical data quite accurately by an analytic function. The result, in any event, is a failure-frequency distribution function. It is then assumed that the failure pattern in the future will be identical with that observed in past tests or operations.

A word of caution is in order to be certain that failure-frequency distribution functions such as those shown previously in Figures 1 and 2 are properly defined. It is common practice to record and tabulate the number of items that fail in a given unit of calendar time and the total number of items operating during that unit of calendar time. Such a representation of failure data is useful for a number of management control purposes, but <u>not</u> for reliability considerations. Except in the special case of constant hazard rate to be discussed later, failure data in this form do not permit valid estimation of the measures of reliability. The data needed, to derive Figures 1 and 2, are not only the number of items that failed and the number operating, but also the age (hours of use) of each item that failed and the age of each item that did not fail. More accurately, then, the failure-frequency distribution should be prepared by tabulating the number of failures as a function of age, rather than by calendar time. This problem usually does not arise in the interpretation of test data where the items are started on tests at the same time.

The observed number of failures in a time interval may be divided by the number of items in the original group to give a failure-rate description of past usage. Almost all work in reliability makes the following (or some equivalent) very important assumption: the observed failure rate for past use or tests is the best estimate of the failure rate to be expected for future usage. Since failure rate can be estimated directly from observed data, all of the concepts related to reliability that are discussed in this paper are stated in terms of the failure rate.

Definitions of Concepts Related to Reliability

Failure Rate. If a group of virtually identical items, n(o), is assumed to have started operation at time t = 0, with some number, n(t), surviving to time t, then the failure rate, Y(t), is defined as the ratio of the number expected to fail in a unit of time Δt at time t to the initial number of units. This quantity is also called the failure-probability density function. In the general case, failure rate is a function of the number of hours the item has operated and is, hence, a function of time in the sense of age of the items. Failure rate is also a function of kind of unit of time chosen as a measure. Failure rate per hour and failure rate per minute are quite different numerically, even though they describe the expected failure rates of the same item.

Thus,
$$Y(t) = \frac{1}{n(o)} \frac{n(t)-n(t+\Delta t)}{\Delta t}, \tag{1}$$

and in the limit as $\Delta t \rightarrow 0$,

$$Y(t) = -\frac{1}{n(o)} \frac{dn(t)}{dt}. \tag{1'}$$

Hazard Rate. The hazard rate is defined as the ratio of the number of failures expected in a unit of time to the number of items of equipment operating during the unit of time. In other words, it is the probability that an item will fail in a given unit of time. In the general case, hazard rate [Z (t)] is a function of the number of time units, e.g., hours, the item has operated and is, hence, a function of time.

Thus,
$$Z(t) = \frac{1}{n(t)} \frac{n(t)-n(t+\Delta t)}{\Delta t}, \tag{2}$$

and in the limit as $\Delta t \rightarrow 0$,

$$Z(t) = -\frac{1}{n(t)} \frac{dn(t)}{dt}. \tag{2'}$$

Hazard rate is frequently used and referred to as a measure of the reliability of an item. It is of use when failures are relatively frequent, and the total maintenance and item-replacement load are of interest. Maintenance incidents for 1000 hours of use and replacement parts required per 1000 hours of use are examples of hazard rates.

Probability of Survival. The probability of survival, S(t), is the probability that a given item will survive from start of operation to some time t. This quantity is also the ratio of the number of virtually identical items, n(t), expected to survive to time t to the number of items, n(o), that started operations at time t = 0.

Thus,
$$S(t) = \frac{n(t)}{n(o)} , \tag{3}$$

$$S(t) = 1 - \int_0^t Y(t)dt . \tag{3'}$$

The probability of an item not surviving, U(t), is then

$$U(t) = 1 - S(t) = 1 - \frac{n(t)}{n(o)} , \tag{4}$$

$$U(t) = \int_0^t Y(t)dt . \tag{4'}$$

The function U(t) is the cumulative failure distribution.

The fraction of a given number of items that may be expected to survive a given number of hours of use is an important consideration in scheduling overhaul facilities and planning the purchase of spares and replacements. This fraction is the survival probability for the number of hours of use of interest.

The half-life may be defined as the number of hours of usage that a new item has a 50 per cent chance of achieving or, by the equivalent definition, as the number of hours of usage at which 50 per cent of a group of items may be expected to survive. Thus, the half-life is that time at which the survival probability, S(t), is one-half.

Relationships. It is apparent that hazard rate, Z(t), failure rate, Y(t), and probability of survival, S(t), are related. The product of Z(t), Equation (2'), and S(t), Equation (3), is equal to Y(t), Equation (1'):

$$Y(t) = Z(t)\, S(t) , \tag{5}$$

$$Y(t) = Z(t)\, [1 - U(t)] . \tag{5'}$$

Examination of Equations (4') and (4) makes it apparent that:

$$Y(t) = \frac{d}{dt}[U(t)] , \tag{6}$$

$$= \frac{d}{dt}[1 - S(t)] . \tag{6'}$$

Hence,
$$Z(t) = \left[\frac{\frac{d}{dt}[U(t)]}{1 - U(t)} \right] . \tag{7}$$

Reliability. Although many different definitions of reliability are in use, one definition is gradually gaining common acceptance as a basis for quantitative statements as to the reliability of machines and equipment.

Reliability is the probability that an item or a system will perform its assigned task for a period of time without malfunction as a function of the assigned task, environment, and previous history.

Reliability is measured in terms of the probability that a given event, a failure, will not occur. However, the terms "perform", "malfunction", and "assigned task" have many possible definitions. Thus, stemming from this single general definition, there are a large number of specific definitions of reliability.

Reliability, $R(t)$, by this definition is the probability of survival over a given period of time t_1 to t_2.

Thus,

$$R(t) = \frac{n(t_2)}{n(t_1)} ,$$

$$= \frac{n(t_2)\ /n(o)}{n(t_1)\ /n(o)} .$$

Using Equation (3):

$$R(t) = \frac{S(t_2)}{S(t_1)} ,$$

or using Equation (3'):

$$R(t) = \frac{1 - \int_0^{t_2} Y(t)dt}{1 - \int_0^{t_1} Y(t)dt} . \tag{8}$$

Mean Time to Failure. If it is assumed that a group of items is operated, or tested, until all have failed, then the mean or average time to failure, T, is the sum of the individual times to failure divided by the total number of items observed.

Thus,

$$T = \frac{1}{n(o)} (t_1 + t_2 + t_3 + \ldots t_{n(o)}) . \tag{9}$$

Since $n(o)Y(t)$ is the number of items failing in an infinitesimal interval of time t,

$$T = \frac{1}{n(o)} \int_0^{\infty} t\, n(o)\ Y(t)\ dt , \tag{10}$$

or

$$T = \int_0^\infty t \, Y(t) \, dt \,. \tag{11}$$

This quantity T is also the expected life of the system.

If each item in the group is replaced, or repaired and returned to service, upon failure, then $\frac{T}{n(o)}$ is the expected interval of time between failures, and T is the expected interval between failures for any one system. Replacement of failed parts is normal to most operations. Hence, T in the context of average time between failures is useful as a guide to estimating maintenance requirements.

Some Specific Failure Patterns

If the data necessary for estimation of the failure rate, the failure-frequency distribution and the number of items in use as a function of age, are available as empirical data, any of the other functions related to reliability can be calculated directly. Such data are available when there is an extensive body of operational history or test experience.

However, for much reliability study, it is necessary to estimate reliability when no body of empirical information is available. It is then desirable to express reliability functions analytically. It has been observed by a number of investigators in the field of reliability that one or the other of the following two analytic representations fits much observed data rather well: constant hazard rates [Z(t) a constant] and a normal or Gaussian failure rate [Y(t) a function of time]. Most real sets of empirical data for complex systems indicate that the best representation of reality can be achieved by combining these two analytic expressions with an empirical "infant mortality" hazard rate in such a manner that each pattern of failure is dominant in a different period in the life of the system. Each of these patterns and the combination of all three patterns are discussed separately in the following sections.

Constant Hazard Rate. The simplest hazard rate is the case in which $Z(t) = a$, a constant. This is equivalent to assuming that an item has an equal chance of failing in any time period. The failures, then, are random in time. Making this substitution in Equations (7) and (4) with $t_o = 0$, $t_1 = t$ gives

$$S(t) = e^{-at} \,. \tag{12}$$

Hence, this failure pattern is frequently described as exponential. Consequently, when a is known, the reliability at any time can be found. The forms of Z(t), S(t), and Y(t) for this failure pattern are shown in Figure 3.

For the exponential law, as is used here, the distribution function becomes $f(t) = N\,a\,e^{-at}$ and mean time to failure becomes

$$T = \frac{1}{N}\int_0^{\infty} N\,a\,t\,e^{-at}\,dt = \frac{1}{a}. \tag{13}$$

Thus, the mean time to failure is the reciprocal of the hazard rate.

In the case of systems in which no replacements are made, the survival probability of the set of systems at $t - T$ is $S = \frac{1}{e} = 0.37$. This indicates that the survival probability of the population at mean time to failure is 0.37, or 37 out of 100 systems survive at $t = T$. Quite often mean time to failure is defined by this fact. Thus, mean time to failure is that time at which $S = 0.37$.

<u>Gaussian Failure Rate</u>. As the system population ages, the components and, consequently, the system may wear out or show evidence of fatigue failures. This happens more or less uniformly to all systems.

The hazard rate, in this case, increases rapidly near the end of life of the population. Since hazard rate is the ratio of failures to survivors, its maximum value is one. The hazard rate due to fatigue failure approaches one near the end of the life of the systems. This observed phenomenon may be approximated by assuming that Y(t) is normally distributed about some mean, b, with standard deviation s. Then:

$$Y(t) = \frac{1}{s\sqrt{2\pi}}\,e^{\left(-\frac{1}{2}\ \frac{t-b}{s}\right)^2}$$

The forms that Y(t), Z(t), and S(t) take for this failure pattern are illustrated in Figure 4. The mean time to failure, T, is by definition equal to b.

<u>Infant Mortality Rate</u>. When a group of systems first begins to operate, some will fail immediately or at least in a very small time interval at the beginning of the operating interval. This is similar to the mortality rate of infants: as time increases, the number of deaths decrease. Because of this resemblance, this type of failure rate is called "infant mortality rate". The form taken by S(t), Z(t), and Y(t) for the infant mortality pattern is shown in Figure 5.

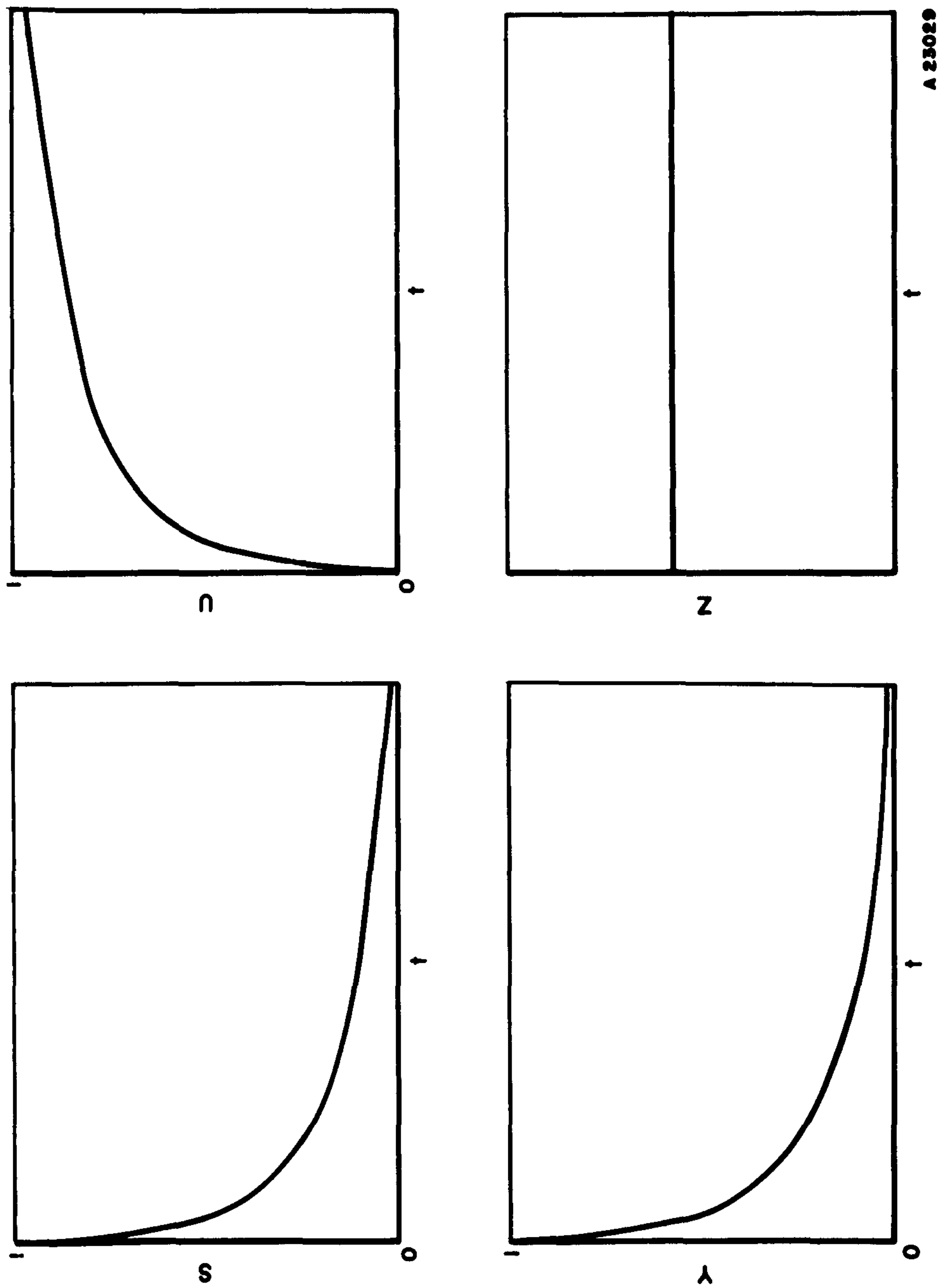

FIGURE 3. EXPONENTIAL FAILURE PATTERNS FOR S, U, Y, AND Z

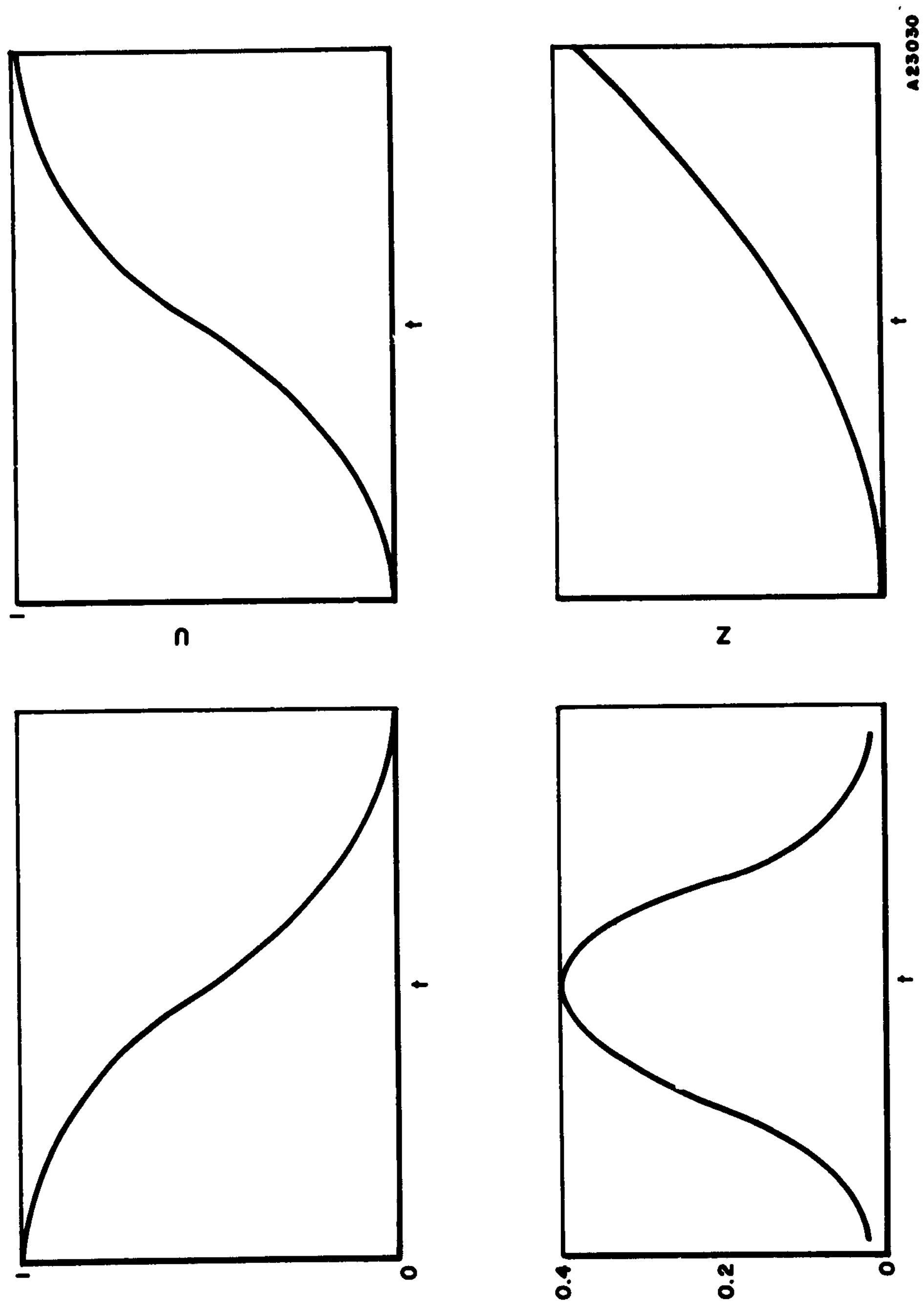

FIGURE 4. GAUSSIAN OR NORMAL FAILURE PATTERNS FOR S, U, Y, AND Z

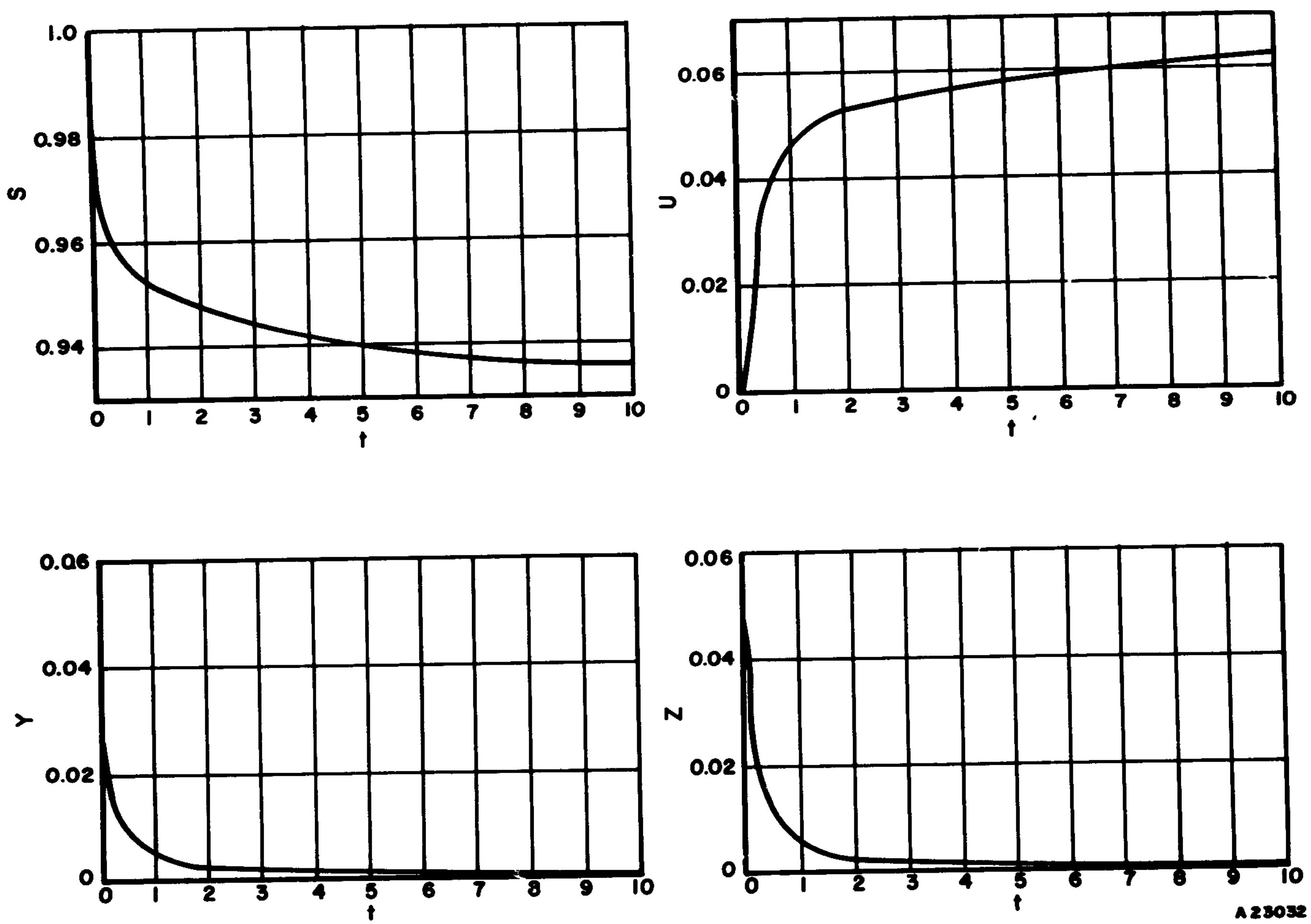

FIGURE 5. INFANT MORTALITY RATES FOR S, U, Y, AND Z

In order to calculate R at any point, Z must be defined. This is usually done by curve-fitting methods.

Application of Reliability Theory to Propeller Controls

In this report, "catastrophic" reliability was defined as the probability that a given propeller system will not exhibit a malfunction in a stated number of operating hours that could lead to catastrophic consequences for the aircraft. It is general practice to state this number in terms of the probability that a catastrophic malfunction will occur (i.e., unreliability); the smaller the number, the more reliable the system.

The definition of "emergency" reliability presented in this report was the frequency of propeller-control malfunctions requiring manual feathering of a propeller by the pilot or other equivalent emergency action. Again, unreliability is used as a yardstick for reliability, and the smaller the number, the more reliable the system.

The fraction of operating controls that reach the mandatory overhaul age without premature removal is a measure of the reliability of the control in the context of the depot-maintenance concept of reliability.

The definition of field-maintenance reliability was stated (in terms of unreliability) as the number of malfunctions that occur per 1000 flying hours that can be corrected by field maintenance; the smaller this number, the greater the reliability of the system.

The first of these, catastrophic reliability, would be represented, as phrased, by the symbol R, as defined earlier in this report. The emergency and field-maintenance definitions of reliability are actually stated in terms of the hazard rate, Z(t). The depot-maintenance definition of reliability is stated in terms related to S(t).

These relationships can best be interpreted by assuming a specific failure pattern. The assumption that hazard rate is a constant, almost certainly a different constant, for each concept of reliability will serve.

Catastrophic Reliability.

$$\text{Assume } Z_c(t) = a \ ,$$

$$R_c = e^{-\int_{t_1}^{t_2} Z_c(t)dt} \ ,$$

$$R_c = e^{-a(t_2 - t_1)} .$$

The probability of a catastrophic malfunction on the next flight is exponentially related to the hazard-rate constant and the duration of the next flight.

Emergency Reliability.

Assume $Z_e(t) = b$,

where $Z_e(t)$ is the expected frequency of emergency malfunctions.

Field-Maintenance Reliability.

Assume $Z_f(t) = c$,

where $Z_f(t)$ is the expected rate of field-maintenance incidents.

Depot-Maintenance Reliability.

Assume $Z_d(t) = d$,

$S_d(t) = e^{-dt}$,

where $S_d(t)$ is the fraction of the initial population expected to survive to time t.

Reliability and Catastrophic Failure

Catastrophic reliability has been defined (Reference 4) as the probability that a propeller control will perform for a period of time without a malfunction resulting in a catastrophic loss of the aircraft. Propeller controls are vital elements of an aircraft upon which safety or flight depends and therefore can be directly responsible for loss of the aircraft in the event of failure. Similar elements affecting safety of flight are the engines, flight controls, and of course the primary structure, such as the wings. It has been pointed out (Reference 31) that an acceptable failure rate for the primary structure is of the order of 1×10^{-7} per hour. This value is also approximately the number of commercial airline catastrophies experienced per flying hour.

The vital elements of structure, power plant, and flight control operate independently of one another. It seems reasonable then to expect the catastrophic failure rate of any one element to be not less than 1×10^{-7} per hour.

Concept of Redundancy

The objective of redundancy is to design equipment to operate satisfactorily in spite of failure of certain critical parts. Reliability of military equipment has been achieved in the past largely through redundant design and high safety factors in strength of materials. Especially in manned aircraft, redundancy is credited (Reference 7) with achieving suitable reliability. Thus, in redundant designs, even though failure may occur relatively often, the ratio of malfunction to catastrophic failure is extremely large, i.e., 10,000 to 1.

Manned aircraft possess inherent redundancy wherever a crew member retains manual control over an equipment function in the event of failure. Only in cases where human capabilities cannot cope with the problem manually, such as the reverse-thrust phenomena in propeller control, has the presence of a human control loop failed to improve reliability significantly with respect to catastrophic failure or mission success.

In general, it can be said that military aircraft design practice has admitted the success of redundancy and encouraged its use. Although not often explicitly stated, the official design guidance in the form of handbooks and specifications tacitly implies the acceptance of redundancy. The "fail-safe" concept of operation is an example. Despite weignt penalties and increased maintenance, redundancy has proved to be a satisfactory solution in many respects to the attainment of high reliability. One bad effect, however, has been to reduce the emphasis on study of the essential characteristics of failure in various applications.

Redundancy has a substantial impact on the observable effects of certain types of failure and, hence, on field testing and evaluation of equipment. For example, in propeller controls, the reverse-thrust situation inherent in most controls is avoided through the use of pitch locks, low-pitch stops, negative torque correction, and automatic feathering devices paralleling the main control elements. Thus, an internal failure that could generate emergency conditions may not be observed. The resulting action, automatic or otherwise, to feather the propeller may not be attributed to the actual cause in later analysis. The same is true for malfunctions resulting in transfer of control from the automatic, self-regulating devices to the pilot. The common reasons given for such action are mildly fluctuating propeller speed or off-speed operation, and these do not give any positive indication of the source or ultimate result of failure.

Catastrophic Failure Events – The Multiengine Case

Redundancy has a significant effect on the consequences of failure and hence on the relative frequency of catastrophic results for a given design. Redundancy has been accepted as an expedient measure in attaining safety of flight. It is widely used in the design of aircraft hydraulic, flight-control, and electronic systems. In the following paragraphs, an attempt will be made to show the effect of design redundancy on propeller-control reliability in the context of the four-engine aircraft configuration.

Catastrophic failures attributable to the propeller control can occur in any of several ways:

(1) Simultaneous failures of an emergency nature, not catastrophic in themselves but in combination, such that thrust from more than two engines is lost

(2) Failure of a single critical component or part such that secondary effects are catastrophic:

- (a) Negative thrust, leading to loss of aerodynamic control of the aircraft
- (b) Vibration leading to primary structural fatigue and failure
- (c) Vibration leading to blade rupture, thrown parts, and possibly fire.

In the first case, consider each propeller control to be one integral unit independently affecting the operation of each power plant. The probabilities of emergency-type failure of the set of four controls involving failures such as loss of governing or hydraulic-valve failure or mechanical failure of the pitch-change mechanism, and others (Reference 4), are defined as follows:

$P_i \quad = P_r \left\{ F_i \right\}$ = probability of a single propeller-control failure

$P_{ij} = P_r \left\{F_i F_j\right\}$ = probability of two simultaneous propeller-control failures

$P_{ijk} = P_r \left\{F_i F_j F_k\right\}$ = Probability of three simultaneous propeller-control failures

$P_{ijkl} = P_r \left\{F_i F_j F_k F_l\right\}$ = Probability of four simultaneous propeller-control failures.

Now, from combinatorial analysis, all possible combinations must be considered and overlapping cases eliminated.

Let $S_1 = \sum P_i$, $S_2 = \sum P_{ij}$, $S_3 = \sum P_{ijk}$, and $S_4 = \sum P_{ijkl}$.

Then the probability, P_1, of at least one failure among all possible events is

$$P_1 = S_1 - S_2 + S_3 - S_4.$$

In the four-engine case, assuming all controls are identical:

$$P_1 = 4P_i - 6P_i^2 + 4P_i^3 - P_i^4.$$

When $P_i << 1$,

$$P_1 \cong 4 P_i .$$

Thus, the probability of at least one emergency failure when four controls are present is approximately four times the probability of failure for a single control.

However, aircraft equipped with four turboprop engines are normally capable of emergency flight operation on only two engines. At least three must be lost before the failure becomes catastrophic.

There are four possible ways that three controls can fail simultaneously. The probability of multiple, simultaneous failure is then

$$P_3 \cong 4 P_i^3 .$$

Approximate values for probabilities of simultaneous occurrence of emergency-type failures are shown in Table 3. To meet the catastrophic reliability requirement of not more than one occurrence of three simultaneous control failures in 10^{-7} hour, the probability of failure for individual controls must be in the range $0.001 < P_i < 0.01$. A specific value of P_i satisfying the equality $P_3 = 4 (P_i)^3$ is 0.00292 per hour.

TABLE 3. PROBABILITY OF SIMULTANEOUS OCCURRENCE OF EMERGENCY-TYPE FAILURES

Number of Engines Lost	Probability of Control Failure		
	$P_i = 0.1$	$P_i = 0.01$	$P_i = 0.001$
1	4.641×10^{-1}	$\sim 4 \times 10^{-2}$	$\sim 4 \times 10^{-3}$
2	6.41×10^{-2}	$\sim 6 \times 10^{-4}$	$\sim 6 \times 10^{-6}$
3	4.1×10^{-3}	$\sim 4 \times 10^{-6}$	$\sim 4 \times 10^{-}$
4	1×10^{-4}	$\sim 1 \times 10^{-8}$	$\sim 1 \times 10^{-12}$

The time dependence expressed here and elsewhere in this section arises, not from any time dependence of the failure function, which is independent of time in the random-failure region, but from the incidence of causes that are time dependent.

In the second case, failure of a critical component causes secondary effects that can be catastrophic, such as negative thrust or vibration damage. This is the case where failure of a single control has catastrophic consequences. In this case, redundancy within the control itself is of primary interest. The basic components in a propeller control are usually considered to be in series; that is, all are required to function without failure if the control is not to fail. The probability of successful system operation in a series arrangement is the product of the probabilities of successful operation of the components. This is often called the "product rule" and is expressed as

$$\text{Probability of system operation} = (P_1)\,(P_2)\,(P_3)\,\ldots.\,(P_n)\ ,$$

where P is the probability of successful component operation and n is the number of components in series. In this sense, the system resembles a chain in which any single element with poor reliability substantially degrades the entire system. The components in such a system would need to have failure rates at least an order of magnitude lower than the required minimum system failure rate. Fortunately, the propeller-control designs presently in existence are not of this sort, but contain both series and parallel components arranged in redundant fashion so as to achieve the high catastrophic reliability level required.

To estimate the reliability of a complex system, a functional definition of the system must be carefully devised so as to show clearly the true relationship of the components in accomplishing the system function. This is a most important first step in the reliability analysis. In Figure 6, several functional diagrams of propeller-control arrangements are shown. In the following paragraphs, the effects of the redundancy illustrated in Figure 6 are discussed.

The basic components in a propeller control, Figure 6a, are the governing mechanism and the pitch-change mechanism. These can be considered to be functionally in series, since both must operate in sequence if the control is to operate successfully. The catastrophic reliability of this system in turboprop applications is particularly sensitive to the component reliabilities, since a failure can lead directly to catastrophic consequences. If each of the two basic components has a probability of failure of 1×10^{-2} per hour, the probability of catastrophic failure approaches 2×10^{-2} per hour. To achieve an acceptable level of reliability, each would need to be developed to the point where probability of failure was less than 1×10^{-8}. If the probabilities of failure were not equal, one probability would need to be even less than 1×10^{-8}. The present state of the art in airborne systems design does not appear capable of producing such designs, now or in the near future.

In actual practice, a form of redundancy aimed specifically at reducing catastrophic failure and increasing safety of flight makes possible the realization of acceptable systems. One example is shown in Figure 6b. By introducing safety devices in the form of pitch locks, low-pitch stops, and negative torque control, along with the inherent redundancy available with the pilot in manned aircraft systems, it becomes possible essentially to parallel all vital components. In the event of failure in the arrangement shown in Figure 6b, there are two backup components for each of the functional operations to prevent catastrophic consequences in the event of component failure.

The reliability of the complex-redundant system in Figure 6b, assuming independent operation for each component, can be expressed as

$$P = [1-(1-P_1)(1-P_2)(1-P_3)]\ [1-(1-P_4)(1-P_5)(1-P_6)] \ ,$$

where P is the component reliability. For purposes of illustration, assume all component reliabilities are equal and the probability of failure associated with each is 0.0030.

Then

$$P = [1-(1-0.997)^3]^2$$

$$= 0.999\,999\,946.$$

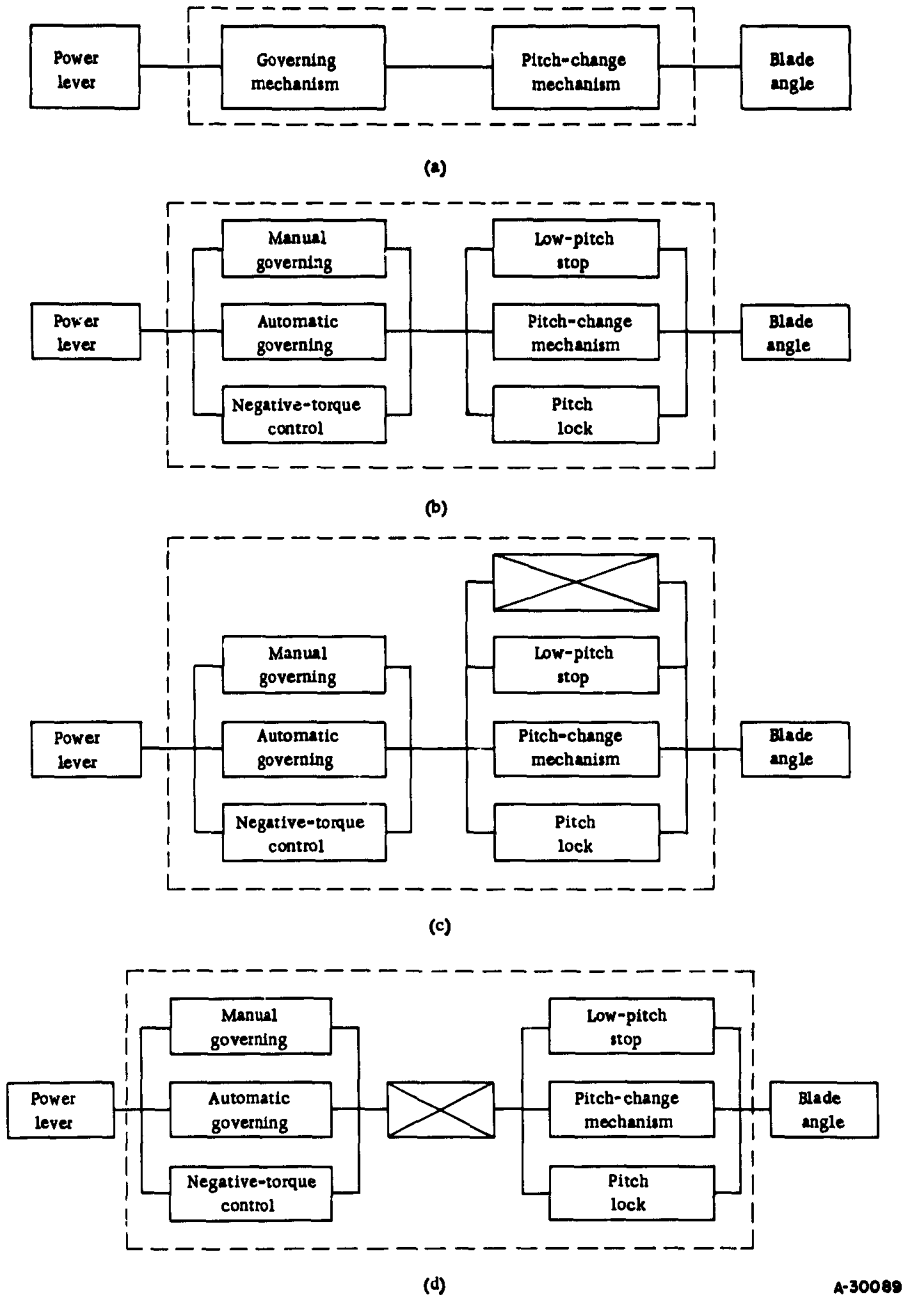

FIGURE 6. FUNCTIONAL DIAGRAMS OF PROPELLER CONTROL

The probability of catastrophic failure of a system for the functional configuration defined in Figure 6b is

$$1 - P = 5.4 \times 10^{-8} < 1 \times 10^{-7} .$$

Thus, the redundant design appears to be acceptable in terms of catastrophic reliability. In this case, where all component reliabilities are equal, it makes little difference how redundancy is effected as long as both vital elements are paralleled by at least two other components.

If each vital component is paralleled only once, the probability of catastrophic failure increases to 1.8×10^{-5}. If no redundancy is employed, the probability of catastrophic failure is 6×10^{-3}. Hence, the functional requirements stemming from safety considerations have also served to provide the necessary margin of reliability in terms of catastrophic consequences.

Obviously, the probabilities of failure for all components in existing propeller controls are not equal, nor is the over-all problem so simple as the example above. However, experience is not too different from the hypothetical case. One would normally expect engineering experience to be sufficient to design most propeller-control components with probabilities of failure less than 0.010, or the equivalent of one expected failure in each 100 hours in the random-failure region. Safety devices and emergency backup components could reasonably be expected to have probabilities of failure less than 0.0010. With these reliability values, the configuration in Figure 6b is estimated to have a probability of catastrophic failure of 1.8×10^{-7}, reasonably close to the desired value.

If the probability of failure of one of the two vital components is reduced to 0.0050, the probability of catastrophic failure decreases to 1.35×10^{-7}. Alternatively, if one additional safety device is inserted paralleling either or both of the vital components, as in Figure 6c, the probability of catastrophic failure decreases a larger amount, to 9.03×10^{-8}.

Should only one vital component be inserted into the functional sequence, Figure 6d, in such a way as to be outside that portion of the system paralleled by safety devices, the effect of redundancy is nullified. Assuming the component reliabilities expected from engineering experience as above, the probability of catastrophic failure for the configuration in Figure 6d would be 1.1×10^{-2}, or just slightly less than the component reliability itself.

Again, in Figure 6b, introducing a switch with a fairly high failure probability, say 0.020, in series with each safety device reduces catastrophic reliability by several orders of magnitude. Assuming the same component reliability as above, based on engineering experience, the probability of catastrophic failure increases to approximately 1×10^{-5}.

Naturally, redundancy does not achieve its desired goal without some associated penalty. From the maintenance standpoint, additional components introduce greater numbers of individual failures and inevitably increase the burden of maintaining the equipment. In view of the relatively low component failure rates required in this type of equipment, however, this does not appear to be a serious problem.

Vital and Critical Components

The terms "vital" and "critical" components have been used to describe the basic elements in the propeller-control system. In view of the above discussion, the following definitions are given:

> A vital component is one that must be paralleled or backed up by one or more safety devices in order to achieve the required minimum probability of catastrophic failure for the system.
>
> A critical component is one for which the catastrophic consequences of failure cannot be overcome through redundant design and that exhibits a conditional probability of catastrophic failure in the event of malfunction greater than 1×10^{-7}.

In the present state of the art in aircraft systems design, vital components are common. The use of redundancy to overcome the tendency toward catastrophic consequences is likewise prevalent in practice. The important steps in evaluating the adequacy of the design are:

(1) Correct functional definition of the system

(2) Determination that redundant components are in fact independent of the basic functional sequence

(3) Determination of the validity of the estimated component reliabilities or failure probabilities.

One critical component can control the incidence of catastrophic consequences in a system. Generally, there is some major design deficiency that must be eliminated from either the basic system design or the component design. For redundant systems encountered in aircraft, the ratio of failure to disaster is probably at least 10,000 to 1 (Reference 7).

For the systems discussed in the preceding paragraphs, this ratio is more nearly 100,000 to 1. One critical component can make catastrophic failure 100,000 times as probable.

The propeller blades, hub, gears, and shafts are prime examples of critical components. Propeller controls are not in the sequence of force transmission that is so difficult, if not impossible, to parallel. Thus, it is unlikely that a critical component need exist in the control design.

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APPENDIX B

RELIABILITY TESTING

APPENDIX B

RELIABILITY TESTING

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APPENDIX B

RELIABILITY TESTING

WADC Qualification Test Evaluation

Description of Tests

In order to provide a basis for suggesting future test modification and standardization, a general study of WADC's propeller-control-system qualification-testing procedures has been made. The major aim of this study was to determine the nature of the information being obtained from the present qualification tests. A statistical estimate of the confidence that can be placed in the control system from the information received has been included.

Test phases that provide information for estimating control-system reliability are listed on WADC-O Form 412 as follows:

(1) Electric Motor Whirl Rig Test

(2) Full Scale Engine Test

(3) Environmental Tests

(4) Functional Tests

(5) Simulator Test

Four prototype control systems are provided for test purposes – one for each of the first four tests. The fifth test involves analog simulation of the system. The remainder of this discussion will consider these five tests.

Electric Motor Whirl Rig Test. In this test, the propeller, with blades installed, is driven by an electric motor. The purpose of the test is to check the functional suitability of the system, that is, to see whether the propeller system performs in accordance with the purchase specification requirements. The test normally includes surveys, calibrations, 20 hours of endurance running, and 1 hour of overspeed running.

Ideally, the information from this test is relevant to the following questions:

(1) Does the system deliver the desired thrust under ideal operating conditions?

(2) Does the system demonstrate the capability to withstand operating stresses under ideal operating conditions?

(3) What failure rate and pattern (infant mortality) are observed during the first few hours of operation?

(4) Are the components of the system compatible? (Conversely, are there any failures because the components do not fit together properly, because of unequal expansion due to heat, etc. ?)

(5) What types of failure (vital, critical, subcritical) are observed?

Simulator Test. In this test, the functions of the system are duplicated by an analog computer. The load and design parameters of the system are varied to determine the limits of operation. These tests are pertinent to the following questions:

(1) What is the time of response of the system to commands and feedback signals? Are the transients, stability, etc., as specified in the design?

(2) What are the theoretical operating limits of the system? (For instance, what is the maximum load under which the control will operate within tolerance?)

(3) What is the theoretical force loading on the major subassemblies?

Data collected here provide a basis for comparing functional suitability observed by physical testing against the theoretical design suitability and, hence, a check on the quality of workmanship employed in the construction of the pilot production.

Full Scale Engine Test. In this test, the entire propeller assembly is driven by its associated aircraft engine under normal bay conditions. The test requires 150 trouble-free hours. However, the test may be run intermittently, with maintenance action allowed. Only a roughly defined "catastrophic" failure requires that the test be rerun. Properly instrumented, this test answers the following questions:

(1) What is the infant mortality rate and failure type (vital, critical, subcritical) under normal bay conditions?

(2) How does the system respond to command and feedback signals under normal bay operating conditions?

(3) How much control has the system over the operation of the engine?

(4) Has vibration affected the operation of the system?

(5) Is the system easily installed and maintained?

(6) Are the components compatible?

Environmental Test. The environmental test ranges from 40 to 50 hours in length. Temperature and pressure (altitude) are cycled in accordance with MIL-E-5272A. Information is obtained pertinent to these questions:

(1) What is the infant mortality rate failure pattern and failure type (vital, critical, subcritical) of a system subjected to temperature and pressure cycling?

(2) Is the system easily installed and maintained?

(3) Are the components compatible?

(4) How is the endurance capability of the system affected by temperature and pressure cycling?

Functional Test. The functional test subjects the system to overstress conditions. The system functions observed at the high stress levels are:

(a) Feathering operation (at overspeed)

(b) Pitch cycling endurance

(c) Operating torques on manual levers (at various levels of overspeed)

(d) Tests of control safety provisions.

The information gathered is relevant to these questions:

(1) What is the failure-rate pattern (infant mortality) and type (vital, critical, subcritical) of a system subjected to overstress conditions?

(2) Is the system easily installed and maintained?

(3) Are the components compatible?

(4) How is the endurance of the system affected by overstress conditions?

(5) How does the system respond to command and feedback signals under overstress conditions?

(6) What are the operational limits of the system? (For instance, what is the maximum overstress condition under which the system will operate within tolerance?)

Conclusions

It is noted that duplication of information occurs in these tests. This duplication is desirable in one sense and undesirable in another.

The major point in favor of duplication of information under various conditions is that correlations can be made that will help determine the effect of a particular condition on a specific system parameter. For instance, a governor assembly might function perfectly under several imposed conditions, but show a number of failures when the system is subjected to pressure cycling. In such a case, the condition that caused the failures could be isolated.

From a reliability standpoint, however, duplication of information under various conditions is undesirable because the hours of test under these conditions cannot be combined to give an over-all system reliability figure. This reliability viewpoint will be discussed in detail in the section devoted to the reliability aspects of the qualification testing problem.

A tabular summary of the results of the preceding study is shown in Figure 7.

Basically, the qualification test subjects the propellers to three conditions: (1) ideal ground conditions, (2) temperature and pressure environment, and (3) overstress. Under the assumption of a true exponential failure rate, this is equivalent to operating a system for A hours under ideal conditions, adding Environment I (temperature and altitude) and operating the system for B additional hours, and finally adding Environment II (overstress) and operating the system for C additional hours. Because of this difference of conditions, A, B, and C cannot be added together and considered as total operating time. However, assurance figures based on these three conditions may be combined to give an assurance figure based on the test as a whole.

			TESTS: Whirl Rig	Simulator	Full Scale	Environmental	Functional
Operational Suitability — Functional Suitability	Theoretical	Command Response		√			
		Feedback Response		√			
		Operational Limits		√			
	Actual	Command Response			√		√
		Feedback Response			√		√
		Operational Limits	√				√
		Engine Control			√		
Environmental Suitability		Temperature				√	
		Pressure				√	
		Vibration			√		
		Overstress	√				√
Maintainability		Frequency of Early Removal	√		√		
		Adequacy of Maintenance Manuals			√	√	√
		Accessibility			√	√	√
Component Compatibility			√		√	√	√
Failures	Pattern	Infant Mortality	√		√	√	√
		Early Abrupt			√	√	
	Type		√		√	√	√

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FIGURE 7. SUMMARY OF QUALIFICATION TEST INFORMATION

To simplify matters, the 10-hour whirl-rig test will be combined with the 150-hour full-scale engine test in what we have called ideal ground conditions. The qualification test breaks down then as follows:

Ideal ground condition	160 hours
Temperature and altitude conditions	50 hours
Overstress condition	500 hours
Total	710 hours

If the four ystems on test are identical and an exponential failure rate is assumed, the qualification testing of four systems is equivalent to testing one system for 710 hours. These hours are divided into three groups, corresponding to the three conditions imposed.

An example of the calculation of assurance based on the present qualification test results should clarify the above statements. Assume that four propeller systems complete the tests with no failures observed. This is equivalent to one unit surviving the three aforementioned conditions with no observed failures. Now, what assurance do we have that the mean time to failure of a typical system exceeds the desired mean time to failure - say 50 hours?

First, the ratio of duration of test to desired mean time to failure is $\frac{160}{50}$ or 3.2 for the normal bay condition. Referring to Figure 8, it is easily seen that the level of assurance corresponding to the r = 0 curve is 95.9 per cent. This, then, is the assurance figure estimated by this test.

Next, the assurance figures for the 50- and 500-hour tests are found from Figure 8. These figures are:

50-hour test: 63.2 per cent
500-hour test: 100 per cent

These assurance figures may then be multiplied together to give a final figure of 60.6 per cent assurance that a typical control system will go at least 150 hours (3 x 50 hours) without failure.

If one or more failures were observed during the test, the assurance figure would be correspondingly lower.

Now, let us suppose for a moment that these three tests can be combined. This would be possible if all three conditions were encountered in

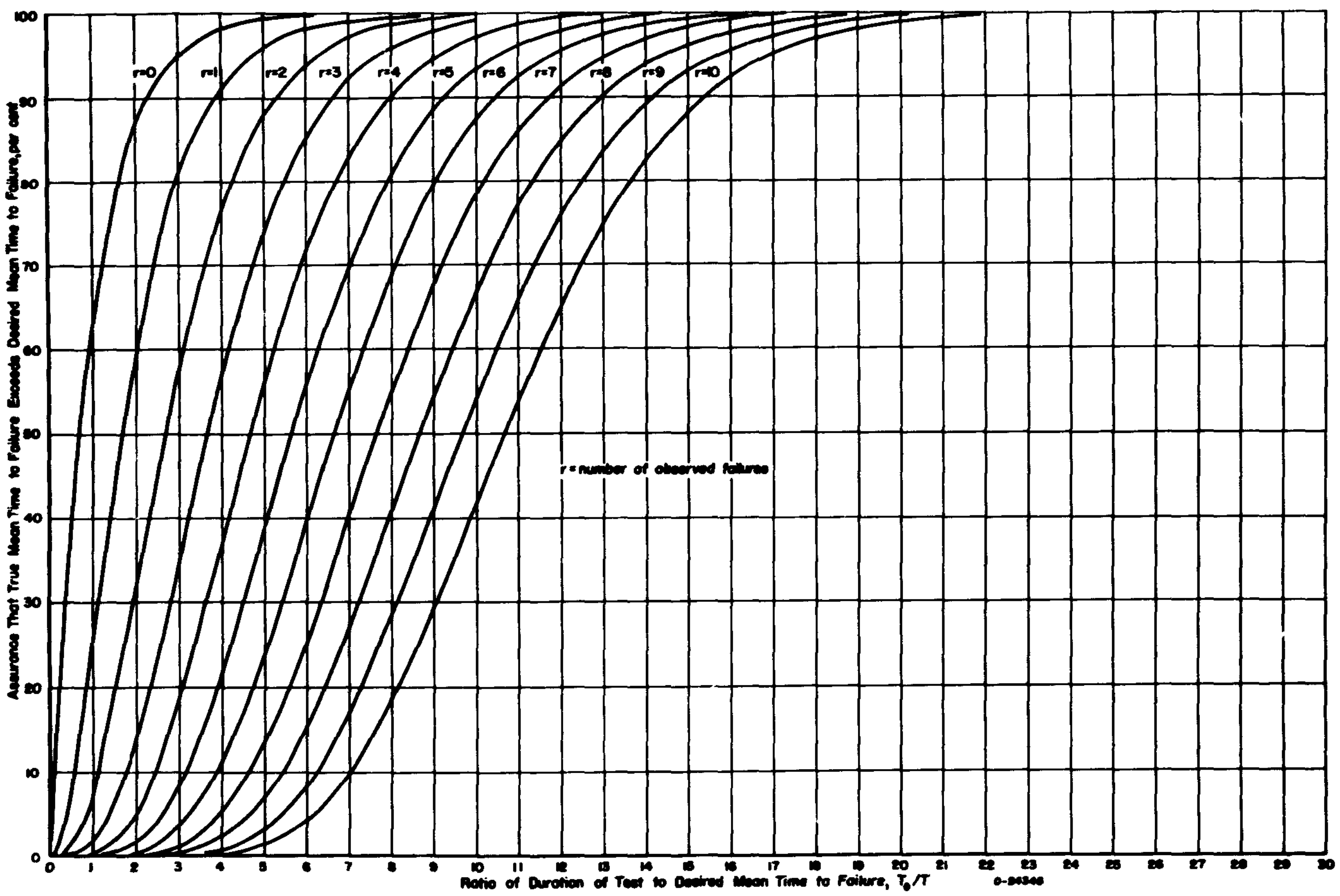

FIGURE 8. LEVEL OF ASSURANCE THAT TRUE MEAN TIME TO FAILURE EXCEEDS DESIRED MEAN TIME TO FAILURE FOR EXPONENTIAL DISTRIBUTION

each of the three tests, or if the individual conditions were weighted and arranged in such a way that the complete qualification test simulated actual flight conditions. The ratio of duration of test to desired mean time to failure is now $\frac{710}{150} = 4.7$ (assuming a desired mean time to failure of 150 hours). If no failures were observed during the test, Figure 8 shows that the assurance of true mean time to failure exceeding the desired mean time to failure is very nearly 100 per cent. Perhaps a more graphic way of showing the advantage of being able to combine the three tests is to compute what desired mean time to failure may be chosen for the confidence level found previously (60.6 per cent) when the test hours were not combined. Figure 8 shows that the ratio of duration of test to desired mean time to failure at 60.6 per cent for no observed failures is approximately 0.92. The mean time to failure for 710 hours of test is then 710 x 0.92 or 653.2 hours, as compared with 150 hours when the test hours cannot be combined.

This study has brought to light two important facts.

First, if the qualification tests were modified so that the data obtained were more valuable from a reliability standpoint, these same data would be less valuable from a specific "cause of failure" standpoint. The system as a whole should perhaps be tested in a manner that would yield the most reliability information and important subassemblies tested separately under isolated conditions for the specific cause-and-effect information.

Secondly, it seems obvious that more data must be obtained if the desired confidence levels are to be predicted or estimated. There are three courses of action that will provide this increased level of information:

(1) Increased sample size

(2) More hours of test

(3) Accelerated testing to simulate increased testing time.

A combination of the above would undoubtedly prove the most satisfactory from both economical and technical standpoints.

Concepts for Revision of the Testing Program

Simulated Use Tests and the Necessity for Test Acceleration

In the study of the present qualification test for propeller-control systems, several facts were brought out. First, the duplication of information from tests under differing environmental conditions is desirable in determining cause-of-failure information, but is undesirable for the purpose of predicting reliability. Second, more complete information must be obtained from the qualification test in order to estimate the life of the control system to the desired reliability levels. A combination of more testing hours and increased sample size would be necessary to obtain the required information. A possible solution to these problems would involve testing the entire control package under conditions simulating actual flight operation. The desirable cause-of-failure and time-to-failure information could be obtained by testing vital components and subassemblies separately under isolated and accelerated stress conditions.

The study of the present qualification test has pointed out that combining test hours from the four systems on test allowed much greater confidence in the reliability estimate than treating each test separately. However, the grouping of nonhomogeneous test data is fallacious. In order to pool such data, the test conditions imposed on each system must be identical, and each system tested must be representative of the final production model. Even if we could control these possible variables, obtaining time-to-failure information from a simulated-use system test would be extremely impractical. The stated requirement for the catastrophic failure rate of the turboprop-control system demands greater than 99 per cent probability of a catastrophic failure rate within 1500 hours of less than 10^{-7} per flight hour. Assuming that no catastrophic failures are observed within the testing period, at a 99 per cent confidence level the necessary test time would be 5×10^7 hours.

Even by pooling data from a large sample size, system testing on such a large scale would be economically out of the question. Similarly, it can be calculated that 1.54×10^5 hours of system tests would be necessary to be 99 per cent sure that the system would have an emergency-action failure rate within 1500 hours of less than 3.25×10^{-4} per flight hour. Again, the cost of this amount of testing on a system level would be prohibitive. The only solution to this testing problem involves some form of accelerated testing. A possible solution lies in the fact that the causes of catastrophic failures and/or emergency-action failures can be isolated and assigned to a relatively small number of components. These components may be of such a nature that tests at high acceleration rates, using large sample sizes, might be applicable.

Twenty-five hundred hours of test would be necessary to be 99 per cent confident that the system would have a maintenance-action failure rate within 1500 hours of less than 0.002 per flight hour. This finally comes within the realm of practical testing. Besides, separate testing of components would not be applicable to predicting maintenance failures usually caused by the failure of subcritical components that make up the majority of system components. For this reason, a test of the entire system is more practical than a test of all the components. The test module for such a system test should be 1500 hours. This would assure staying in the operating life of the system where the exponential failure rate applies and supplying the information necessary to fulfill a reliability requirement that 50 per cent of these systems must survive 1500 flight hours without premature removal. Two methods for conducting such a test are:

(1) Subject all propellers on test to a mixture of all conditions expected in normal flight operations

(2) Subject the systems on test to isolated conditions arranged and weighted so as to simulate a flight cycle.

The second method, in addition to providing the necessary reliability information, would allow correlations to be made that would help determine which imposed conditions caused particular types of failures.

Some of the facets of the system testing problem that should be examined are:

(1) What conditions should be imposed on the system and what limits should be placed on these conditions?

(2) Is it desirable to attempt to accelerate the system test?

An obvious answer to the first part of question one is, "those conditions that are encountered in the operational environment". Defining this operational environment, however, poses a problem. An aircraft may be operated in Alaska or in Panama. It may be flown daily or only occasionally. It may or may not be flown in smooth weather the majority of the time, depending upon the location of the home base. Thus, not one, but many possible environments exist. A practical laboratory test, however, could involve only one environmental cycle. Since the control system has to operate satisfactorily at any location, the qualification test should include the environmental extremes – for instance, the high temperature of Panama and the low temperature of Alaska. This line of reasoning leads to two conclusions:

(1) A practical test cannot be devised that will simulate a real operational environment.

(2) The most logical choices for the test limits are the operating environment extremes, but this possibly introduces an unknown acceleration factor, since the test would of necessity be more stringent than any one actual environment.

The term "environment" as used in this discussion includes any condition or influence, natural or human, that affects the control system. Thus, such factors as weather condition, overstress, vibration, and maintenance action are considered part of the environment. Ideally, a complete system test should include all of the conditions encountered in an operating environment. However, such a test conducted in the laboratory would be impractical.

The second question pertains to the advisability of accelerating a system test. System deterioration can be accelerated by increasing the stress imposed. However, it would be extremely difficult to determine just what acceleration factor had been introduced, since the degree of acceleration would be different for each component and subassembly in the system. A great deal of operational data would be necessary in order to compute acceleration factors, and this volume of data is usually not available on a prototype control system submitted for qualification testing.

Another factor to be considered is the relationship between failures and stress level. There is always a specific cause for every system failure. If the mode and the distribution of failures remained the same when the stress level was increased, acceleration might be feasible; but it does change. This fact is most easily depicted by referring to the most commonly used S-N plot for fatigue-failure data. Points signifying failure occurrence for a given stress level are plotted in a rectangular-coordinate system, where stress level is the ordinate and the number of stress applications is the abscissa. The distribution of failures can be roughly approximated by two straight lines, as shown in Figure 9. These two segments are referred to as the "derating curve" and the "infinite" or "near-infinite" life curve. The statistical implications of these two segments are totally different. The derating curve is plotted through the mean of the distribution of failure occurrences at various stress levels, whereas the infinite life curve is rather poorly defined and is constructed through observations of the first failure.

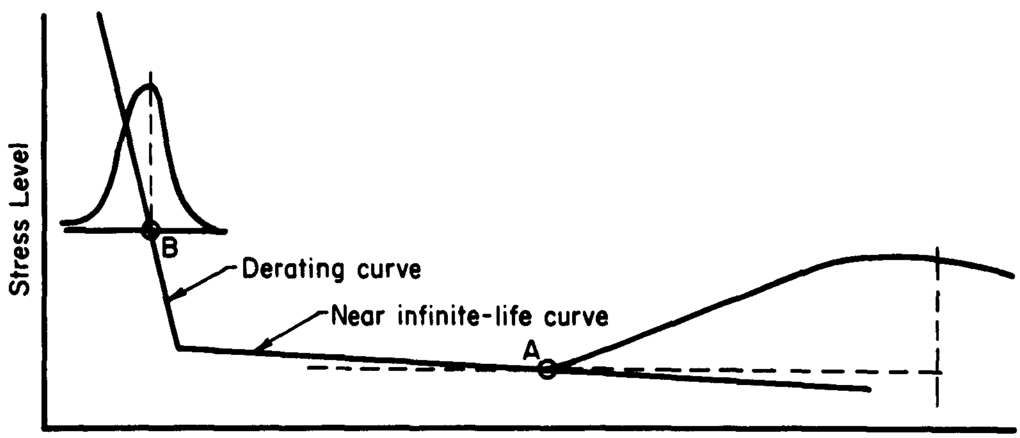

FIGURE 9. IDEALIZED S-N PLOT

This plot basically applies only to components, but since the plot of a system is a composite of the combined S-N plots of its components, the plot shown is valid for a system in a general discussion such as this. Assume that the normal operating stress occurs at point A. If the stress is increased to point B in an attempt to accelerate time, the operating point moves up onto the derated portion of the curve. On this part of the curve, the assumption that the mode of failure is the same as that under normal stress no longer holds. This can be explained by the fact that, although a normal distribution of failures probably occurs at both stress levels, point A is on the tail of this distribution, whereas point B is somewhere near the mean of a normal distribution. The fact that at stress level A the failures of interest lie at the end of the distribution allows the assumption of an exponential failure rate. This assumption is not valid at stress level B and cannot be called a true acceleration of time. Why not accelerate the number of stress applications, then? This is possible in the case of, say, a level that normally is activated a finite number of times during the normal operation of the system. With such components as shafts, however, an increase in the stress level is experienced as the shaft is rotated at an accelerated rate. There may be a solution to this problem without having to use component testing, but it would involve unique instrumentation and would probably be economically impractical.

From a consideration of these two problems, it appears that the most practical method for predicting maintenance failure rates would involve field tests. Laboratory tests cannot hope to simulate the multitude of environmental conditions that will be encountered in actual field use. Also, any form of simulation runs the risk of imposing undue stress conditions on the system, thereby distorting the test results. This distortion would not be a problem if we had some means for determining what it was. However, the quantification of this distortion factor would involve the collection and analysis of a great quantity of field-operation and laboratory test data.

The Problems of Accelerated Testing

As previously stated, catastrophic and emergency-failure rates cannot be determined by simulated-use tests. Since the minimum required test duration is 10^8 hours, some means must be found to accelerate time in order to accomplish the desired testing in a practical length of time.

The basic assumption in the formulation of accelerated testing techniques is the analysis of the relationship between applied stress and the number of applications of that stress. These two factors are the major contributions to fatigue failures, and it is the fatigue failure that is the most difficult to predict with any degree of confidence. Normal attrition or wear can be studied in the laboratory by radioactive techniques and the "wear curves" extrapolated to predict the end of life with considerable precision. Up to the present time, the study of the S-N relationship has been used only as a theoretical technique to aid the designer in the fabrication of reliable equipment. To our knowledge, the implications of this relationship for the evaluation of the reliability of hardware has never been considered.

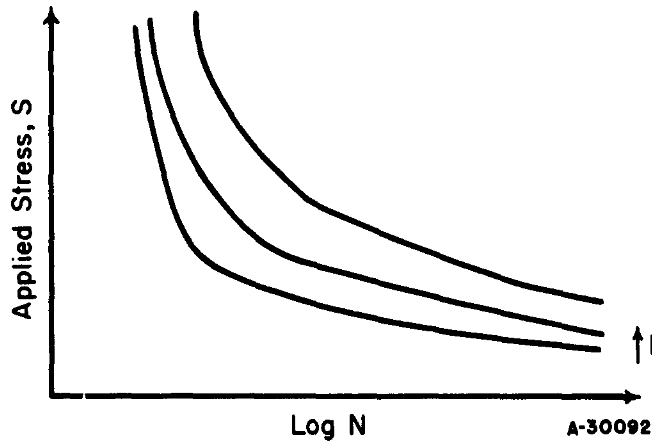

FIGURE 10. S-N PLOT

Figure 10 indicates the relation between stress applications, S, and the number of cycles of application of the stress for various failure percentiles. These curves are considered as representing three regions: (1) the derating region, associated with a relatively small number of applications of a relatively large stress, (2) the infinite life region, associated with a large number of applications of a small stress, and (3) an intermediate region, or transition region, between the derating and infinite life regions. In the fatigue failure of materials, these regions generally have distinct physical interpretations.

An analysis of the S-N relationship leads to the following conclusions:

(1) To accelerate the life properties of any element accurately so that the results of the accelerated life test are directly proportional to the actual life of the element under normal operating conditions, the stress level imposed in testing must not be raised above the maximum stress level experienced under normal operating conditions. The only variable that can affect acceleration without introducing distortion is the frequency of application of applied stress. If the stress level is raised, the degree of distortion imposed on the test results must be quantified in order to make the results meaningful.

(2) The exact nature of the stresses imposed on the various components must be analyzed in order to determine just how much acceleration may be introduced before the stress level begins to rise.

(3) The approximation of the S-N distribution by a pair of straight lines is extremely rough. Actually, the transition from the infinite life portion of the "curve" to the derated portion is a gradual one.

(4) If the stress level is raised by the introduction of a large acceleration factor, there is no real justification for assuming that the added stress is of the same type as the normal stress.

(5) If the stress level is raised by acceleration, the calculation of an acceleration fa or becomes complicated by the addition of a second variable. If the normal operating stress is maintained, the acceleration rate is merely a simple time relation that is easily determined.

(6) In Figure 11, the maximum time limit for the accelerated test module occurs at t_L, where the constant-hazard-rate (exponential failure rate) portion of the curve ends. Now, modify the time axis by the ratio f_n/f_o, where f_o is the normal activation frequency of a component and f_n is some accelerated frequency. Then f_n/f_o is the acceleration factor. Assuming that the stress level is not affected by an increase in frequency to, say, f, the curve of Z versus $t(f_n/f_o)$ for $f = f_1$ would be identical to the curve for $f = f_o$. Now suppose the frequency were increased to some frequency f_2 that does cause an increase in the stress level. The curve will now be altered in some fashion; probably, the constant hazard rate will be raised, the Gaussian pattern changed, and T_L shifted. At least, some noticeable change would be expected. Some frequency f_2 will be the limiting activation frequency. At frequencies above f_L, the curves of Z versus $t(f_n/f_o)$ begin to differ.

The preceding statements present a specific framework within which the reliability of components and systems may be studied. However, strict adherence to S-N theory places severe restrictions on the design of test conditions and the implementation of accelerated tests. In order to design a practical accelerated test, the fatigue theory must be preserved in essence, but incorporated into some more general approach that will enable a relaxation of the requirement for absolute adherence to actual use stress conditions. A further objection to strict adoption of the S-N theory of failure arises from the assumption that a periodic stress is the sole contribution to the fatigue of the components materials. Although such a contention may prove to be correct for turbopropeller control systems, it may be expected that the reliability of some systems would not be appropriately treated in terms of periodic stresses. In particular, nonperiodic or static stresses may be of more importance in producing failures in certain systems and may contribute greatly to the generation or propagation of material defects involved in fatigue phenomena.

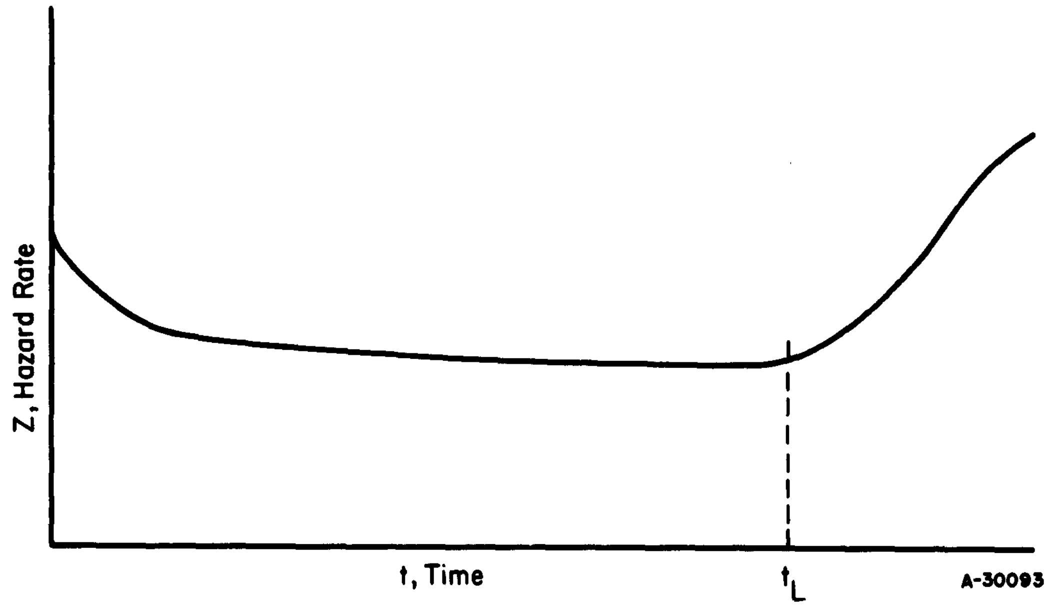

FIGURE 11. NORMAL SYSTEM FAILURE PATTERN (HAZARD RATE VERSUS TIME)

Moreover, the very existence of the S-N curve suggests that fatigue phenomena are not expressible in terms of material properties. For example, the objective of a mathematical formulation of fatigue failures would be an expression having the form:

$$p = f(S, N, X_1, \ldots, X_n),$$

where p denotes the probability of failure, S denotes applied stress, N denotes the number of stress applications, and $X_1, \ldots, X_n$ denote material properties such as tensile strength, hardness, element configuration, etc. Because it is common practice to omit the material properties, one is led to suspect that the material properties have never been satisfactorily related to fatigue failures. Thus, a statistical description is all that remains in the usual S-N curve, based on

$$p = f(S,N),$$

where the curve is valid only for the particular element studied. This suggests that true modeling of fatigue failures in dissimilar elements is not possible, a conclusion that is also substantiated by the following remark by Langhaar[1]:

> "There are some fields in which dimensional analysis has had little application, because the existing knowledge in these fields is inadequate to indicate the significant variables. For example, the endurance limits of members that are subjected to alternating stresses have not been correlated with other measurable properties of materials. Consequently, dimensional analysis (and the associated theory of models) cannot yet be brought to bear on questions of fatigue of materials."

These remarks suggest that if, in fact, reliability is describable as a fatigue problem, and if the material properties are not strongly related to fatigue failures, then the statistical description typified by the S-N curve may be the only practical approach to the acceleration problem.

To maintain its validity, the S-N relationship requires that applied stress be described in terms of environmental forces or conditions. In the more generalized formulation, the probability of failure thus has the form:

$$p = f(S^*,N),$$

where

$$S^* = g(Y_1,\ldots, Y_n)$$

and $g(Y_1,\ldots, Y_n)$ denotes some function of environmental conditions and S^* serves as a measure of the "severity" of the generalized stress. In a similar manner, the meaning of the number of stress applications becomes more general when the generalized stress is introduced. Basically, the number of generalized stress applications is a measure of time. Thus, the formulation of the failure probability may be written as

$$p = f(S^*, N^*),$$

where

$$S = g(Y_1,\ldots, Y_n)$$

and

$$N^* = h(t).$$

(1) Langhaar, H. L., Dimensional Analysis and Theory of Models, John Wiley & Sons, Inc., 1951, p 15.

The S-N curves are thus replaced by S*-N* curves, which may exhibit properties quite similar to those of the S-N curves. However, interpretation and meaning of the generalized stress and time function may be more appropriate to the reliability problem.

The use of model theory appears to be a practical solution to resolving these objections. By proper applications of accepted modeling techniques, a suitable scaling factor may be introduced such that distortion appearing in accelerated test results may be quantified. In addition, generalized stresses may be considered and material properties taken into account.

APPENDIX C

ACCELERATED TESTING AS A PROBLEM OF MODELING

APPENDIX C

ACCELERATED TESTING AS A PROBLEM OF MODELING

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ACCELERATED TESTING AS A PROBLEM OF MODELING

INTRODUCTION

The prediction of catastrophic failures of a highly reliable system is often unconvincing when made without benefit of actual data. Moreover, because excessively long time periods are required to obtain a single catastrophic failure, the reliability of such a system is often beyond direct experimental inquiry.

In this case, a turboprop control system is required to have a malfunction rate with catastrophic consequences of less than 1 in 10 million flying hours. The experimental demonstration of the fulfillment of such a requirement with a newly produced turboprop control system is extremely difficult. Neither the manufacturer nor the purchaser of such highly reliable equipment can afford the time required to amass the actual experience that will make the reliability computations valid. The problem becomes even more pressing when the purchaser refuses the equipment unless the required reliability is first established by the manufacturer.

These problems indicate that suitable methods are needed for the prediction of the reliability of highly reliable systems. In addition to experimental and theoretical validity, these methods should yield reliability predictions that are more firmly established than calculations made without the benefit of actual data. Such methods should also yield reliability predictions within reasonably short time periods for systems that are highly reliable. It is the purpose of the following development to indicate that the theory of models may sometimes furnish methods and criteria suitable for the rapid generation of experimental data that may be used to predict the probability of a catastrophic failure of a highly reliable system. Reliability prediction based on models represents a compromise between unconvincing reliability calculations based on paperwork estimates and calculations based on actual operational experience.

The importance of modeling in engineering is well known. In many instances of advanced technology the construction of a prototype is never attempted until the performance characteristics of models are determined. Models of airframes, ship hulls, and dams are sometimes geometrically similar to their prototypes, so that the model appears to be uniformly reduced or expanded in size. In flow of fluids, geometrical similarity is often exchanged for dynamic similarity, such that the observed forces in the model are directly proportional to corresponding forces in the prototype. In some instances no direct dimensional ratio need exist between model and prototype, as evidenced by the use of analog computers to simulate electronically the characteristics of nonelectronic systems. Many examples of practical applications of modeling theory have been given by Langhaar[1], Birkhoff[2], and others[3,4].

In most applications of modeling, the physical performance characteristics of the prototype are predicted from data obtained from experimental studies of the corresponding characteristics of the model. Such characteristics usually consist of drag, mass flow, temperature, etc. To a large extent the only novel feature of the present development is the assertion that the same general method of modeling is also valid for the prediction of prototype reliability when using the observed reliability associated with a model.

The use of models to obtain data for the prediction of the reliability of the prototype may also raise difficult problems. Some of the assumptions, criteria, advantages, and difficulties associated with a model-theoretic approach to reliability prediction are examined in this study.

BASIC CONCEPTS IN THE THEORY OF MODELS

Dimensional Analysis

The theory of modeling rests on the principle of dimensional analysis, which, in turn, rests on certain assumptions regarding the mathematical formulations of physical relations. These assumptions have been summarized in mathematical form by Birkhoff.[2] In descriptive terms, it is assumed (1) that all physical quantities can be measured in terms of appropriate combinations of fundamental units, such as lb/in.^{2}, ft/sec^2, (2) that the magnitudes of physical quantities measured in terms of one set of units may be re-expressed in terms of a new set of units by the familiar process of "changing units", (3) that a physical equation expresses one "dependent" variable as a function of several "independent" variables, and (4) that the validity of such a function does not depend on the particular set of fundamental units chosen. Although these assumptions are generally valid for laws of physics and mechanics, the philosophical implications and interpretations have been in dispute since the time of Fourier[5] when this method was first formulated.

As a rigorously deduced consequence of the assumptions associated with dimensional analysis, Buckingham[6] proved the so-called π-theorem. The symbol π refers to "products" of the original variables used to describe the performance of a physical system. In descriptive terms, Buckingham proved that the dependent and independent variables in a mathematical function involving physical variables can be replaced by appropriate products and ratios of the original variables. The new variables have the property that their magnitudes are independent of the system of units used, and, consequently, are said to be "dimensionless". Familiar examples of such dimensionless quantities are specific gravities of liquids, radian measures for angles, Reynolds number in fluid flow, Mach number, etc. In addition to the fact that the reformulation of a physical law in terms of dimensionless variables usually reduces the number of variables involved, it is particularly important that Buckingham's π-theorem places a condition on the form of the variables involved in a function, even though the form of the function may not be known.

In view of the above results, a procedure based on dimensional analysis for determining an unknown functional relation involving physical variables is usually given as follows: (1) make a list of the relevant variables, together with their associated dimensions in any consistent system of units, (2) combine these variables in products and ratios to form a complete set of independent dimensionless parameters, and (3) determine the unknown function by experimentally varying the values of the independent dimensionless parameters and measuring the value of the associated dependent dimensionless parameter. The appropriateness of this procedure for the determination of functions giving failure probabilities is examined in the next section.

The Formulation of Reliability Functions

The general problem of predicting the reliability of a physical system has many features that suggest the appropriateness of the techniques of dimensional analysis. For a particular system, it is usually (tacitly) assumed that the probability of failure of the system is expressible as some (unknown) function of relevant variables. For convenience, such a function is called a reliability function. For an electronic system, for example, it may be supposed that the probability of failure in time t, say p(t), is mathematically expressible as a function of operating voltage V, relative humidity H, operating temperature θ, and operating hours T; so that $p(t) = f(V, H, \theta, T)$, where f denotes a reliability function whose form is unknown. It is assumed that such a function exists, and that the form of the function does not depend on whether the operating voltage is measured in volts or millivolts, whether operating time is measured in hours or years, etc. (fourth assumption under "Dimensional Analysis").

The difficulties associated with these assumptions are apparent. The effects of environmental conditions may be more relevant to producing a failure than the internal physical properties of the system. Consequently, the physical system considered must include the environment, the operator, maintenance schedules, and whatever else is believed to be relevant to the prediction of reliability. When formulated in such broad terms, it is clear that enormous difficulties are involved in choosing a set of relevant variables. An appropriate set of variables usually involves those variables associated with the internal processes of the system, the external processes in the environment, and the interaction between the system and its environment.

In a particular problem it is usually not possible to assert that (1) all of the variables included in the functional form are relevant and (2) all of the relevant variables are included. Either error may have serious consequences. If some particularly relevant variable is omitted, the mathematical formulation may be invalid. If irrelevant variables are included, the "noise" caused by their presence may mask the actual significance of the relevant variables. However, there is nothing involved in this formulation that is peculiar to the problem of determining a reliability function. The same problem of choosing relevant variables also exists in the physical sciences in attempting to describe physical processes in mathematical terms. Whereas physical processes are ordinarily described in terms of internal system variables, the reliability process must generally be described in terms of both internal and external variables, and, in some instances, the external, or environmental, variables may be far more important for the prediction of reliability.

If it is asserted that no relevant set of variables exists, then the mathematical formulation of a reliability function is not possible. In a particular context, this assertion is by no means logically invalid. On the other hand, if it is assumed that the probability of failure can be predicted quantitatively, it appears that one must simultaneously assume the existence of a set of relevant variables and the existence of the associated function. Consequently, the assumptions usually involved in dimensional analysis must also be assumed to hold together with the assertion of Buckingham's π-theorem. These arguments thus lead to the following conclusion: If the probability of failure of a given physical system can be mathematically formulated, then the predicting function is expressible in terms of dimensionless power products of the relevant variables. The procedure for obtaining a mathematical formulation is then identical to that based on dimensional analysis given in the preceding section.

The Theory of Models

The assumptions of dimensional analysis lead to Buckingham's π-theorem, which, in turn, yields a quantitative basis for the theory of models. To show this, suppose it is desired to predict the value of a particular variable, say Q_o, associated with the prototype system using experimental data obtained with a model system. The relevant variables for the prototype system are listed, together with their dimensions, in some consistent set of units. These variables are then combined to form dimensionless parameters, $\pi_o, \pi_1, \ldots, \pi_n$, where π_o is a dimensionless combination of variables that involves Q_o. The formal relation among these dimensionless variables may then be written as follows:

$$\pi_o = f(\pi_1, \ldots, \pi_n),$$

where the function f is unknown. A model is then constructed that involves a corresponding set of dimensionless parameters π_o', $\pi_1', \ldots, \pi_n'$, and a formal relation among them:

$$n_o' = f'(n_1', \ldots, n_n').$$

If the model is so constructed that $\pi_1' = \pi_1, \ldots, \pi_n' = \pi_n$, and if it is assumed that the functional form of f' is identical to that of f, then it follows that $\pi_o = \pi_o'$, and this equation may be easily solved for the value of Q_o for the prototype. Thus, the basic idea involved in a model-prototype relation may be briefly described as follows: The corresponding independent dimensionless parameters of the model and prototype are made numerically equal. Then, under the assumption of identical functional forms among these independent variables, the corresponding dependent dimensional parameters must also be equal. From the equality of the corresponding dependent variables, predictions of prototype performance can be made using the observed performance characteristics of the model.

To illustrate the preceding results and to introduce further terms associated with this model-theoretic approach to reliability, it is convenient to consider the familiar assumption of an exponential distribution of failure times. In its simplest form, the assumption states that the probability density function p(t) for the failure time of a given system is given by:

$$p(t) = \frac{1}{\tau} e^{-t/\tau}, \ t > 0,$$

where the constant τ denotes the mean life of the system. It is apparent that, other than elapsed time, none of the physical properties of the system are assumed to be relevant to the prediction of failure. Note that (t/τ) is a dimensionless parameter so that, provided t and τ are measured in the same time units, it makes no difference whether seconds, hours, or days are adopted as units of time. Moreover, the "element of probability", p(t)dt, which gives the probability of failure of the system in the time interval (t, t + dt) may be written as:

$$p(t)dt = e^{-t/\tau} \, d(t/\tau),$$

where both (t/τ) and the differential, $d(t/\tau)$, are dimensionless. When written in this form, the exponential failure assumption conforms with the conclusion derived earlier that any mathematical formulation of the probability of failure of a given physical system is expressible in terms of dimensionless ratios and products of the relevant variables. It is easily shown that other commonly used distributions, such as the Normal distribution and the Poisson distribution, also satisfy the dimensionless requirement.

Under the assumption of an exponential distribution of failure times for the prototype, the theory of models shows that any other system that also has an exponential failure distribution may serve as a model for the prototype provided the corresponding dimensionless ratio of the model and prototype are made numerically equal. In this example, a model may be chosen such that:

$$t'/\tau' = t/\tau,$$

where the primed and unprimed symbols refer to the model and prototype, respectively. This equation shows that a model having a mean time to failure that is smaller than that of the prototype will also have smaller failure times, and also shows that the prototype time is related to the model time as follows:

$$t = \left(\frac{\tau}{\tau'}\right) t' .$$

It follows, for example, that, if the ratio of prototype mean time to failure to model mean time to failure is 100, then 1 hour of model operation is equivalent to 100 hours of prototype operation. Consequently, the observed failure probabilities in model operation during time periods of 1 hour are numerically equal to the predicted failure probabilities for the prototype for time periods of 100 hours.

In terminology usually associated with the theory of models, the constants τ and τ' are called "characteristic times", and the ratio (τ/τ') is called a "time scale factor". The ratio (t/τ) may be regarded as an "intrinsic time"; it measures in dimensionless terms the operating time of the prototype in units of the characteristic time of the prototype. More generally, the variables associated with a prototype system are usually examined to determine whether there exist various characteristic measures that, when used as units of measure, will allow all of the variables of the system to be expressed as intrinsic variables. Such intrinsic variables are always dimensionless, and thus suitable for modeling applications.

As in the above example, each equality between corresponding dimensionless parameters of the model and the prototype yields a scale factor. Thus, a model may have various scale factors, one for time, one for length, etc., each of which serves to relate the model to the prototype. In practice, the model characteristics are usually selected to yield scale factors of desirable magnitudes.

In many applications of model theory it is not possible to make the dimensionless parameters of the model exactly equal to the corresponding dimensionless parameters of the prototype. The resulting disagreement between the model and prototype is usually called a distortion, and empirical methods are required to relate the performance of the model to that of the prototype. Generally, several models are found to be necessary for both the dimensionless parameters and the functions that relate them. There is no limit to the type or magnitude of the correction that may be introduced, provided the prototype performance can be adequately and reproducibly predicted on the basis of the performance of the model. However, in such applications, a gradual and painstaking development of appropriate models is usually required.

Accelerated Testing

In order to obtain reliability predictions based on experimental data, it is usually necessary to obtain failures, and for highly reliable systems the time required to obtain failures is excessive. Under these conditions, it is sometimes suggested that the system be operated under "severe" conditions, at higher stresses, higher temperatures, etc., in order to produce failures in a reasonably short period of time. Such "accelerated" testing generally causes the system to fail in shorter time periods than under normal use. However, before such an approach can be adopted, it must be asked: What criteria can be used to relate the failure patterns observed in accelerated tests to the predicted failure patterns under normal use? It is shown below, when certain requirements are met, that model theory can furnish the desired criteria.

The general methods that permit the model-prototype relation to be described for systems of different types will also be valid for systems of the same type. Although trivial, this observation permits the application of the principles of modeling to two identical systems, one of which undergoes accelerated testing (the model), in order to predict the reliability of the same type of system operated in normal use (the prototype).

Underlying the concept of accelerated testing is the assertion that the probability of failure is a function of "stress" and time. As noted earlier, the reliability of a system generally depends on both internal variables and external, or environmental, variables. Consequently, the probability of failure of the prototype system may be written as:

$$p = f(S, T),$$

where

$$S = g(\gamma_1 \ldots, \gamma_n),$$

and $g(\gamma_1, \ldots, \gamma_n)$ denotes some function of environmental and internal conditions. Thus, the "generalized" stress, S, serves as a measure of the "severity" of the operating conditions over a corresponding period of "generalized" time, T. More detailed consideration is given to the concepts of generalized stress and time in a later section.

From Buckingham's π-theorem, it follows that the above probability may be expressed in terms of a dimensionless stress parameter, σ, and a dimensionless time parameter, τ, to yield:

$$p = f(\sigma, \tau). \tag{14}$$

For the model, the probability of failure is written as:

$$p' = f'(\sigma', \tau'), \tag{15}$$

where the model is operated with:

$$\sigma' = \sigma \tag{16}$$

and

$$\tau' = \tau \tag{17}$$

and a time-scale factor that makes 1 model hour equal to a sufficiently large number of prototype hours. Then, under the assumption that the functional form of f' is the same as that for f, so that:

$$f = f', \tag{18}$$

it follows that:

$$p = p' . \tag{19}$$

Thus, the probability of failure for the prototype is equal to the observed probability of failure for the model in a prototype time interval obtained from the model time interval by using the time-scale factor obtained from the equation $\tau = \tau'$. The assumptions of Equations (14), (15), and (18), together with conditions of Equations (16) and (17), are sufficient to obtain Equation (19), which, in turn, relates the probability of failure obtained from accelerated test data to the probability of failure for the system under normal use by means of the time-scale factor.

Although this development shows that model theory yields a suitable framework for the description of accelerated testing, the argument is quite general. To be of practical significance, the generalities involved in the concepts of generalized stress and generalized time and the manner in which stress and time are rendered nondimensional must be considered. The following sections contain more detailed discussions of these matters. After these discussions, the example of accelerated testing is reconsidered at a more practical level.

Concept of Generalized Stress

A generalized stress may include mechanical forces, electrical stresses, thermal stresses, effects due to extremes of humidity, frequency of system operation, and stresses due to periods of storage. In broad terms, the generalized stress may be considered to be the net effect on the system of the internal and external environment at any given time.

Some of the environmental stresses may be regarded as analogous to the "stress application" associated with studies of metal fatigue failures. These stresses include frequency of operation, rotation of shafts, periodic electrical impulses, etc. More generally, those stresses that are periodic in time may be identified with the stress application of metal fatigue studies. The various periods need not be equal nor commensurate. Moreover, in some cases it may be possible to "superimpose" the various periodic phenomena to obtain an over-all characterization of the periodic stresses involved in the system.

In addition to the periodic stresses, there may also be nonperiodic stresses. These include permanent stresses, such as progressive chemical changes, corrosion, static loading, etc., and are associated with gradual wear of mechanical parts, gradual shifting of electrical characteristics, etc. These nonperiodic stresses form a continuous background on which the periodic stresses are imposed.

These considerations lead to the following decomposition of the environmental stresses. The environmental stresses are regarded as composed of periodic stresses superimposed on a background of nonperiodic stresses. The over-all combination of these stresses constitutes the generalized stress, S. A practical problem exists in attempting to obtain a quantitative measure that reflects the magnitude of the over-all generalized stress.

It should be noted that a third component of the environmental stress that is not considered is the "impulsive" stress, that is, a stress induced by an isolated incident in time. Such incidents as a brief mechanical overstress, or an accidental jolt, for example, may create conditions that are important components of the environmental stress. However, unless such stresses occur with sufficient frequency to define a statistical distribution, it does not appear possible to account for these stresses by a model-prototype relation.

A characteristic stress associated with periodic stresses may be defined in any of various ways. The maximum stress amplitude and the root-mean-square amplitude are typical measures suitable for a characteristic stress. For nonperiodic stresses, the adoption of a characteristic stress is more difficult, and an arbitrary definition is usually required.

Concept of Generalized Time

Because of the meaning ascribed to generalized stress, it is clear that the corresponding measure of time may be measured variously as the age of the system, the number of system performances, the number of cycles of shaft rotation, etc. The most appropriate measure of time must be decided for each specific application.

Associated with the periodic components of the generalized stress is a "natural" time, given by the actual time required to complete 1 cycle. The time to complete 1 cycle may be taken to be a characteristic time, and the generalized time of the system is equal to the ratio of real time to the characteristic time. If several periodic components are present, the characteristic time may be taken to be the time to complete 1 cycle of the superimposed components.

For the nonperiodic components of the generalized stress, the choice of a time measure is much more difficult. If a characteristic unit of time can be determined, then the generalized time of the system is given by the ratio of the real time to the characteristic time, as in the case of periodic phenomena. However, because of the absence of a "natural" time unit, somewhat artificial characteristic times are usually associated with nonperiodic phenomena. As an example, the characteristic time associated with the exponential distribution of failures is that time required for the probability of successful operation to be reduced by the factor $1/e$. In general, the appropriate time measure must be decided separately for each problem.

The Problem of Scale Factors

In addition to the practical difficulties in obtaining suitable numerical measures for generalized stress and generalized time, a difficulty of greater subtlety is associated with the choice of scale factors. Having suitably defined the characteristic generalized stress and characteristic generalized time, a model must be constructed with characteristic stresses and times that give desirable scale factors. For example, it may be desired that 1 hour of model operation be equivalent to 1000 hours of prototype operation. Although characteristic times may be obtained that yield this ratio, it may occur that, by choosing such large scale factors, the functional forms f and f' of Equation (18) are no longer identical. Moreover, because the functional form of f is not known, the lack of identity that may exist between these functions is also unknown. On the other hand, it is known that, for scale factors sufficiently close to unity, the model and prototype are practically identical systems, and there is little reason to suspect a lack of identity between f and f'.

One method for partially avoiding this problem is the following. Several tests are made on models that have progressively altered scale factors. The functions associated with the sequence of models are then examined to determine whether the functions change with changes in scale factor. If the functions do not change, then the relation between the model and the prototype is better established. If the functions do change, it is sometimes possible by extrapolation to obtain the function for the prototype.

The difficulties discussed thus far indicate that enormous problems exist in the determination of reliability functions based on data obtained in accelerated tests. However, it should not be concluded that, because of these difficulties, model theory is an inappropriate method of attack. Instead, the conclusion to be drawn is that, through model theory, the difficult problem of accelerated testing can be correctly formulated, and consequently, can be attacked more sensibly.

MODEL-PROTOTYPE RELATIONS SUITABLE FOR ACCELERATED TESTING

It has been shown in the preceding sections that accelerated testing may be regarded as a modeling problem in which it is assumed that the probability of failure is a function of the application of a generalized stress over a generalized time. In addition to the validity of Equations (14) through (19), various assumptions may be made regarding the relations between the characteristic stresses and characteristic times of the model and prototype. If it is assumed that the characteristic stresses are equal, and the characteristic time for the model is smaller than that of the prototype, then a model-prototype relation suitable for accelerated testing is obtained.

Let S'_o and S_o denote the characteristic stresses of the model and prototype, respectively; and let T'_o and T_o denote the corresponding characteristic times. Then the mathematical description of the model-prototype relation is given by the following:

$$S_o = S'_o \tag{20}$$

and

$$T_o > T'_o, \tag{21}$$

together with Equations (14) through (19). By writing Equation (16) in the form:

$$\sigma = \frac{S}{S_o} = \frac{S'}{S'_o} = \sigma',$$

it follows from Equation (20) that:

$$S = S'. \tag{22}$$

Thus, the generalized stresses in the model and prototype are numerically equal. Similarly, by writing Equation (17) in the form:

$$\tau = \frac{T}{T_o} = \frac{T'}{T'_o} = \tau',$$

it follows that:

$$T = \left(\frac{T_o}{T'_o}\right)T', \tag{23}$$

so that the prototype time, T, is obtained from the model time, T', by multiplying the model time by the scale factor (T_o/T'_o). By Equation (21), this scale factor is larger than unity, so that 1 hour of model operation corresponds to more than 1 hour of prototype operation. Relative to the prototype, the time of the model "flows faster", and, consequently, the model may be said to be "accelerated".

In order to ascribe practical significance to this theoretical formulation, it is necessary to obtain a characteristic time that is smaller for the model than for the prototype and, at the same time, maintain equal stresses in the two physical systems. Because an intentional "weakening" of the model structure may destroy the equality of the stresses between the model and the prototype, it would appear desirable to use identical structures for both model and prototype. Under this restriction, changes in characteristic times can be obtained only by altering the use of the two systems in time. Moreover, a system that is in operation most of the time would appear to be ideally suited for modeling an identical system that is operated only occasionally. This is particularly true for systems in which the internal stresses are more relevant to reliability than the environmental stresses. It is also clear that, for some systems, the actual number of operating hours is not the most relevant measure of time. A better measure may be given by the number of hours of operation under those "abusive" stresses that cause significant "degradation" of the system. Unless the system is operated continuously under abusive stresses, the "effective" number of hours of operation would be less than the actual number.

To make these notations more definitive, a physical system is considered for which the following assumptions hold:

(1) The stresses imposed on the system during normal operation can be classified as being either abusive or nonabusive.

(2) The probability of failure of the system is dependent only on the cumulated effects of the abusive stresses over time.

(3) The proportion, A, of the total operating time for which the system is subjected to abusive stresses is known.

Under these assumptions, a characteristic time may be defined as follows:

T_o = number of operating hours required to accumulate 1 hour of abusive stress.

Because A is equal to the ratio of cumulated abuse hours to the total operating hours, it follows that:

$$T_o = 1/A.$$

Thus, the characteristic time may be defined to be the reciprocal of the abuse ratio, A. By increasing the abuse ratio, A', for the model, the characteristic time for the model can be decreased, as shown by:

$$T_o = \frac{1}{A} > \frac{1}{A'} = T_o'. \qquad (24)$$

Equations (23) and (24) then show that the time-scale factor relating the prototype time to the model time is given by (A'/A); that is,

$$T = \left(\frac{T_o}{T_o'}\right) T' = \left(\frac{A'}{A}\right) T'. \qquad (25)$$

Thus, the prototype time is equal to the model time multiplied by the abuse ratio, A'/A. Equation (25) also shows that the maximum scale factor, and consequently the maximum "acceleration", is obtained when A' = 1.0. Under this condition, the model accumulates 1 hour of abuse for every hour of operation, and the prototype-model times are related by the simple expression:

$$T = \left(\frac{1}{A}\right) T'. \qquad (26)$$

This result shows, for example, that, for a prototype system that accumulates 1 hour of abuse for each 100 hours of operation, A = 1/100, and the "most accelerated" model yields 1 hour of model operation, which is equivalent to 100 hours of prototype operation.

For highly reliable systems, it may be permissible to assume that the abuse ratio, A, is very small, so that many hours of operation are required to accumulate 1 hour of abuse. This means that, for such systems, 1/A is very large, so that modeling of such systems in reasonably short time periods appears theoretically feasible. However, the potential difficulties associated with large scale factors require that this conclusion be accepted with caution.

As a result of the highly instrumented tests made on the prototype system, it is assumed that a specific set of operating "states", say $C_1, \ldots, C_n$, is found to be abusive. All those operating states that are found to be nonabusive may be collectively labeled C_o. The abuse ratio is then given by the fraction of the total operating time spent in any one of the states $C_1, \ldots, C_n$ when the system is operated in normal use.

To make the model-theoretic approach practical, it is necessary to describe normal use in quantitative terms. This may be done, for example, by determining the fraction of the total operating time the prototype system is expected to exist in each of the states $C_o, C_1, \ldots, C_n$ when operated in normal use. Let $T^{(i)}$ represent the number of hours spent in state i out of T operating hours, so that $T^{(i)}/T$ represents the fraction of the total operating time spent in state C_i. Similarly, let the corresponding fraction for the model be denoted by $T^{(i)'}/T'$. The objective in operating the model is to minimize the time spent in the nonabusive state, C_o. The time normally spent in state C_o should be distributed over the abusive states in proportion to the fractions $T^{(i)}/T$. More specifically, let the marginal distribution of operating time over the abusive states be represented by $T^{(i)}/\sum_{j=1}^{n} T^{(j)}$, $i = 1, \ldots, n$ for the prototype, and by $T^{(i)'}/\sum_{j=1}^{n} T^{(j)'}$ for the model. These distributions are made identical by the requirement:

$$\left[\frac{T^{(i)'}}{T'}\right]\left[\frac{T'}{\sum_{j=1}^{n} T^{(j)'}}\right] = \left[\frac{T^{(i)}}{T'}\right]\left[\frac{T}{\sum_{j=1}^{n} T^{(j)}}\right], \quad i = 1, \ldots, n.$$

Solving this equation for $T^{(i)'}/T'$ yields:

$$\left[\frac{T^{(i)'}}{T'}\right] = \left[\frac{\sum_{j=1}^{n} T^{(j)'}/T'}{\sum_{j=1}^{n} T^{(j)}/T}\right]\left[\frac{T^{(i)}}{T}\right],$$

and by definition of the abuse ratios, it follows that:

$$\frac{T^{(i)'}}{T'} = \left(\frac{A'}{A}\right)\left(\frac{T^{(i)}}{T}\right), \; i = 1, \ldots, n.$$

This result shows that the fraction of the total operating time spent by the model in state C_i is obtained from the corresponding fraction for the prototype by multiplying by the abuse ratio, A'/A.

To obtain the time spent by the model in state C_o, the above equation is first summed over i to yield:

$$\sum_{i=1}^{n} \frac{T^{(i)'}}{T'} = \left(\frac{A'}{A}\right) \sum_{i=1}^{n} \frac{T^{(i)}}{T},$$

which may be written as:

$$\left[\frac{T^{(o)'} + \Sigma T^{(i)'}}{T'} - \frac{T^{(o)'}}{T'}\right] = \left(\frac{A'}{A}\right)\left[\frac{T^{(o)} + \Sigma T^{(i)}}{T} - \frac{T^{(o)}}{T}\right],$$

so that:

$$\left[1 - \frac{T^{(o)'}}{T'}\right] = \left(\frac{A'}{A}\right)\left[1 - \frac{T^{(o)}}{T}\right].$$

This expression may be written as:

$$\left(\frac{T^{(o)'}}{T'}\right)\left[\frac{T'}{T^{(o)'}} - 1\right] = \left(\frac{A'}{A}\right)\left[\frac{T}{T^{(o)}} - 1\right]\left(\frac{T^{(o)}}{T}\right),$$

and, from $T^{(o)}/T = 1 - A$ and $T^{(o)'}/T' = 1 - A'$, it follows that:

$$\left(\frac{T^{(o)'}}{T'}\right)\left[\frac{1}{1 - A'} - 1\right] = \left(\frac{A'}{A}\right)\left[\frac{1}{1 - A} - 1\right]\left(\frac{T^{(o)}}{T}\right),$$

and finally,

$$\left(\frac{T^{(o)'}}{T'}\right) = \left(\frac{1 - A'}{1 - A}\right)\left(\frac{T^{(o)}}{T}\right).$$

Thus, for a given set of abuse ratios, A and A', the fraction of the total operating time spent by the model in state C_o is obtained by multiplying the corresponding fraction for the prototype by $(1 - A')/(1 - A)$.

The efficiency of the model-prototype relation may be defined by $E = (A' - A)/(1 - A)$. For $A' = A$, the efficiency of the model is zero, and for $A' = 1$, the efficiency of the model is 100 per cent. The time spent by the model in state C_o is expressible in terms of the efficiency of the model, as shown by the relation:

$$\frac{T^{(o)'}}{T'} = (1 - E) \frac{T^{(o)}}{T}.$$

Thus, when the efficiency is zero, the model spends just as much time as the prototype in the state C_o; when the efficiency is 1.0, the model spends no time in the state C_o.

Prediction of Catastrophic Failures Using Model Theory

The preceding results may be applied to the problem of predicting catastrophic failures. To be specific, suppose it is desired to determine whether a turboprop control system meets a reliability requirement that it have a mean malfunction rate with catastrophic consequences of less than 1 in 10 million flying hours. Suppose, further, that such control systems are to be used as models in an accelerated test program. The preceding development of model theory suggests the following approach to the problem:

(1) Establish for the control systems the characteristic time, T_o, the number of operating hours required to accumulate 1 hour of abuse under normal operating conditions. A highly instrumented experimental run to estimate wear rates, temperatures, voltages, etc., may furnish this information for various operating conditions.

(2) Operate a set of model control systems under those conditions found above to be abusive until the first catastrophic failure occurs at time T'.

The aim in Step (1) is to determine the characteristic time, T_o, and, hence, the abuse ratio, $A = 1/T_o$; the aim in Step (2) is to obtain one catastrophic failure of a model system operating with an abuse ratio A' as close as possible to the theoretical maximum of 1.0. The prediction of the catastrophic failure rate for the prototype system operating under normal conditions is then obtained under the model-prototype assumptions for accelerated models given in Equations (14) through (21), and by the time-scale factor relation given in Equation (25). As an example, suppose the abuse ratio, A, is found to be 0.001, so that 1 hour of abuse is accumulated for every 1000 hours of operation. Then suppose that 10 models are simultaneously operated at abuse ratios $A' = 0.90$, so that 9 hours of abuse are accumulated in every 10 hours of operation for each model. The time-scale factor for this case is then given by $A'/A = 0.90/0.001 = 900$, so that 1 hour of model operation is equivalent to 900 hours of prototype operation. Finally, suppose that, among the 10 models on test, the first failure occurs after 1000 hours of operation. The model-prototype relations then show that this is equivalent to observing 1 failure out of 10 prototype systems operating under normal conditions at the end of $(900)(1000) = 9 \times 10^5$ hours. Because this is the smallest failure time out of 10 operating units, the next problem is to use this information to estimate the reliability associated with the population of prototype control systems.

Let f(t) denote the unknown probability density function for failure times associated with control systems operated under normal conditions. The probability density function for the smallest failure time in a set of n observed failures is given by:

$$g(t) = nf(t)\,[1 - F(t)]^{n-1}, \tag{27}$$

where F(t) denotes the cumulative density function associated with f(t). The cumulative density function associated with g(t) is then given by integration of g(t) from 0 to t, as shown by:

$$G(t) = \int_0^t g(u)\,du = \int_0^t nf(u)\,[1 - F(u)]^{n-1}\,du,$$

and integration of the right side yields:

$$G(t) = -[1 - F(u)]^n \Big|_0^t ,$$

so that:

$$G(t) = -1 + [1 - F(t)]^n.$$

Solving for F(t), there results:

$$F(t) = 1 - [1 - G(t)]^{1/n}.$$

Now G(t) represents the area to the left of t in the distribution of smallest failure times among n failure times and may be called a p^{th} fractile. Similarly, F(t) represents the area to the left of t in the distribution of failure times within the population of control systems, and may be called a P^{th} fractile. Thus, the fractiles of the two distributions are related, as shown by:

$$P = 1 - [1 - p]^{1/n}. \tag{28}$$

Figure 12 shows a plot of P as a function of n for p = 0.50 and p = 0.90. The figure shows, for example, that, with a sample of size 10, the probability is 0.50 that 7 per cent of the future observations will be

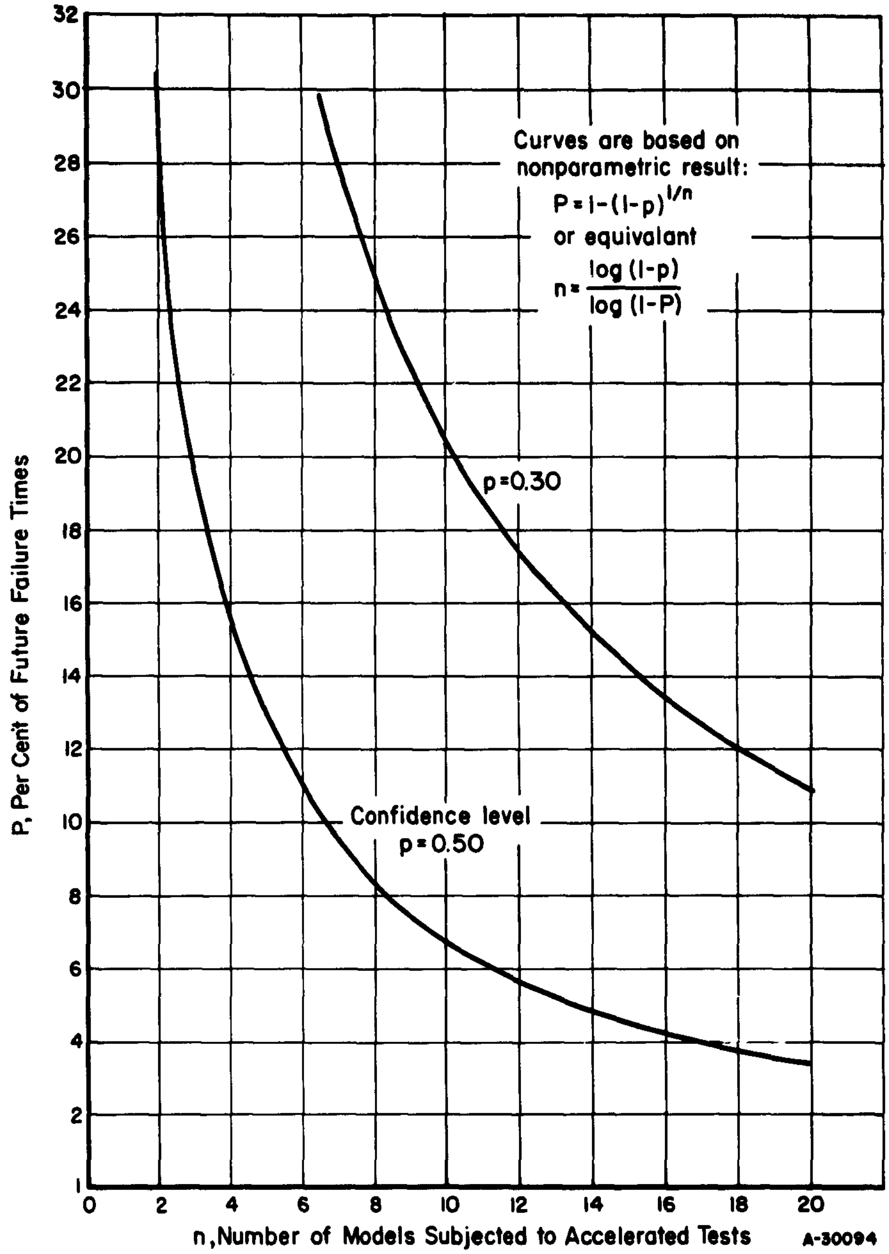

FIGURE 12. ESTIMATED PER CENT OF FUTURE FAILURE TIMES EXPECTED TO BE LESS THAN THE SMALLEST OBSERVED FAILURE TIME

The probability is p that less than P per cent of future failure times will be smaller than the smallest deserved failure time among a set of n models subjected to accelerated tests.

smaller than the smallest observation in a sample of size 10. Similarly, the probability is 0.90 that 21 per cent of the future observations will be smaller than the smallest observation in a sample of size 10. It should be noted that no assumptions involving the forms of the distributions are made in this formulation.

In the example of the turboprop control system, it may be concluded with 50 per cent confidence that the probability is less than 0.07 that a randomly selected control system will have a failure time smaller than 9×10^5 hours; it may be concluded with 90 per cent confidence that the probability is less than 0.21 that a randomly selected control system will have a failure time less than 9×10^5 hours.

In some instances, it may be possible to terminate the accelerated tests before catastrophic failure occurs for one of the models. As an example, suppose the reliability requirement takes the following form: It must be demonstrated with 50 per cent confidence that the probability is less than 0.05 that the system will suffer a catastrophic failure within 10^6 operating hours. Suppose the model and prototype abuse ratios are given by $A' = 0.90$ and $A = 0.001$, respectively. Then from Equation (25) it follows that, to obtain the equivalent of 10^6 operating hours with the prototype system, the models must be operated without a failure for:

$$T' = \frac{T}{(A'/A)} = \frac{10^6}{900} = 1,100 \text{ hours.}$$

The number of models to test is obtained either from Figure 12, or from Equation (28), by solving for the value of n corresponding to given values of p and P. The solution yields:

$$n = \frac{\ln (1 - p)}{\ln (1 - P)},$$

and for $p = 0.50$ and $P = 0.05$ for the example, it follows that n is approximately equal to 14. Thus, if 14 models are subjected to accelerated tests with an abuse ratio of 0.90 for 1,100 hours of operation without a failure, then the tests may be terminated and it may be concluded with 50 per cent confidence that the probability is less than 0.05 that the prototype system will suffer a failure within 10^6 operating hours. If the reliability requirement had been stated as a confidence level of 0.50 that there is less than one chance in a 1000 of a failure in 10^4 operating hours and with an abuse ratio of 900, as above, then 11.1 hours of testing on each of 700 models would be required.

It may be noted that the preceding conclusions do not give estimates of the mean life of the control systems. In particular, the question of whether the control system meets a reliability requirement of an average failure rate of less than 1 in 10^7 operating hours has not been answered in the example. Instead, it has been concluded with 90 per cent confidence that less than 21 per cent of the control systems will have failures in fewer than 9×10^5 operating hours. It is believed, however, that the conclusion illustrated in this example may have more practical value than a conclusion concerning mean life. The reason for this belief is shown by the following argument. A mean-life estimate yields a measure of the "location" of a distribution in the sense of a center of gravity and generally does not indicate the "spread" or dispersion of the distribution about its center of gravity. However, for the prediction of catastrophic failures, the primary concern is with that "tail" of the distribution that contains early failures. Estimates of interest to the prediction of catastrophic failures should give the "size" of this tail. This is precisely the form of the conclusion obtained for the turboprop example. With 90 per cent confidence, the "size" of the tail consisting of failures that occur within 9×10^5 operating hours is estimated to be less than 21 per cent of the control systems. When the size of the tail is very small, say 1 per cent with 95 per cent confidence for the number of operating hours of interest, it may be asserted that the reliability of the system is very high, even though the form of the failure distribution and its mean life are not known.

The Use of the Exponential Distribution of Failure Times in the Model-Theoretic Approach to Reliability

In the preceding sections, the form of the distribution of failure times was not assumed. However, the incorporation of this type of assumption into the model-theoretic formulation is straightforward. Some of the results derived from the assumption of an exponential distribution of failure times are given here.

In the application to catastrophic failures, n failure times are not actually observed. Instead, after the first catastrophic failure has occurred among the n models, the tests are stopped. It is easily seen, however, that the following argument remains valid, because the last (n - 1) failure times are not used in estimating mean life.

The probability density function for an exponential distribution of failure times for the models is given by:

$$f(t') = \frac{1}{\tau'} e^{-t'/\tau'}, \quad t' > 0, \tag{29}$$

where the constant τ' denotes the mean time to failure. Integration of this expression between 0 and t' yields the cumulative density function:

$$F(t') = 1 - e^{-t'/\tau'}, \quad t' > 0,$$

and substitution of these two expressions into Equation (27) yields:

$$g(t') = \frac{n}{\tau'} e^{-nt'/\tau'}, \; t' > 0. \qquad (30)$$

This result represents the probability density function for the smallest failure time in a set of n failure times drawn from an exponential distribution. The expected value of the smallest failure time is given by:

$$E\{t'\} = \int_0^\infty t' g(t') dt' = \int_0^\infty \frac{nt'}{\tau'} e^{-nt'/\tau'} dt',$$

and integration yields:

$$E\{t'\} = \frac{\tau'}{n}. \qquad (31)$$

Thus, an unbiased estimate of the mean life for the models is given by:

$$\tau' = nT',$$

where T' is the smallest observed failure time among the n models on test.

As shown in Equation (25), prototype time is obtained from model time by multiplying the model time by the scale factor, A'/A. The following argument shows that, if the model failure times are assumed to be exponentially distributed, with a mean life of τ', then the prototype failure times are also exponentially distributed, with a mean life of τ' (A'/A).

The element of probability corresponding to Equation (29) is given by:

$$f(t')dt' = \frac{1}{\tau'} e^{-t'/\tau'} dt', \; t' > 0.$$

Elimination of t' and dt' using:

$$t = (A'/A)t',$$

and

$$dt = (A'/A)dt'$$

yields the transformed element of probability,

$$g(t)dt = \left(\frac{1}{\tau'}\right) e^{-(1/\tau')(A/A')t} \left(\frac{A}{A'}\right) dt, \; t > 0,$$

which may be written as:

$$g(t)dt = \frac{1}{\tau'(A'/A)} e^{-t/\tau'(A'/A)} dt, \; t > 0.$$

This result has the form of an exponential distribution, and shows that the prototype failure time is exponentially distributed, with a mean life given by:

$$\tau = \tau'(A'/A). \tag{32}$$

Equation (31) may also be used to obtain an unbiased estimate of the mean life of the prototype system, for:

$$E\{nt'\} = \tau',$$

and, consequently,

$$E\left\{n\left(\frac{A'}{A}\right)t'\right\} = \left(\frac{A'}{A}\right)\tau' = \tau.$$

This result shows that an unbiased estimate of τ, the mean life of the prototype systems is given by:

$$\tau = n\left(\frac{A'}{A}\right)T', \tag{33}$$

where T' is the observed time to failure among a set of n models, and (A'/A) is the time-scale factor.

Estimation of the Probability of Catastrophic Failures for Various Operating Time Periods

By combining the assumption of an exponential distribution of failure times with Equation (28), estimates of the probability of catastrophic failures for various time periods can be obtained. To show this, assume that the first model failure occurred after T' hours. Then the corresponding time of failure for the prototype is T'(A'/A), and, under the assumption that the prototype failure times are exponentially distributed, with a mean life of τ, it follows that:

$$P = 1 - \exp \frac{-T'(A'/A)}{\tau}, \tag{34}$$

where P is obtained from Equation (28) for a fixed value of p. This equation may be solved for τ to obtain:

$$\tau = n\left(\frac{A'}{A}\right)T'\left[\frac{1}{\ln\left(\frac{1}{1-p}\right)}\right]. \tag{35}$$

Comparison of this result with Equation (33) shows that the estimate of mean life given above is biased unless $1/(1 - p) = e$, for which $p = 1 - 1/e = 0.63$. For values of p greater than 0.63, the estimate given in Equation (35) will be "conservative" in the sense that the mean life of the prototype system will be underestimated. Thus, if p denotes the "confidence" in the mean life conclusions, then confidence increases as the mean life is increasingly underestimated.

The probability of failure for the prototype in T operating hours is given by:

$$P_T = \frac{1}{\tau}\int_0^T e^{-t/\tau}\,dt,$$

where t denotes prototype time. Integration yields

$$P_T = 1 - e^{-T/\tau},$$

and substitution for τ using Equation (35) yields:

$$P_T = 1 - (1 - p)^{1/n^*}, \tag{36}$$

where

$$n^* = \left(\frac{A'T'}{AT}\right)n. \tag{37}$$

A comparison of this result with Equation (28) shows that n^* may be regarded as an "effective" sample size for the model tests. The product $A'T'$ represents the total number of abuse hours accumulated by the model before it failed at time T'. Similarly, the product AT represents the total number of abuse hours accumulated by the prototype in T hours. It should be noted that A', T', and A are fixed quantities, whereas T may be varied.

Equation (37) may also be written as:

$$n^* = \frac{(T'/T'_o)}{(T/T_o)}\,n,$$

in accordance with Equations (24) and (25), where T'_o and T_o denote the characteristic times of the model and prototype, respectively. This result shows that the effective number of models is multiplied by the ratio of the intrinsic model time to the intrinsic prototype time.

As an example of the use of Equations (36) and (37), suppose the abuse ratio for the turboprop control system is determined to be 0.001 under normal operation, so that 1 hour of abuse is accumulated for every 1000 hours of operation. Suppose further that 10 models are operated at abuse ratios of 0.90 and the first model failure occurs after 1000 hours of operation. It is desired to estimate the probability of failure for the control system operating under normal conditions for 10^6 hours. Under these conditions, $A = 0.001$, $A' = 0.90$, $T' = 1000$ hours, $T = 10^6$ hours, and $n = 10$, so that:

$$n^* = \frac{(0.90)(1000)}{(0.001)(10^6)} (10) = 9.$$

Thus, the "effective" number of models is 9, and Figure 12 may be used to determine P_T using a sample size of 9. This procedure shows that P_T is approximately 0.22 with 90 per cent confidence. Thus, it is concluded with 90 per cent confidence that the probability of failure of the prototype system when operated for 10^6 hours under normal conditions is equal to 0.22. By repeating this computation for various values of T, the failure probabilities of the prototype system over a range of operating times may be obtained.

A GENERAL MODEL-PROTOTYPE RELATION FOR ACCELERATED TESTING

In the simple model-prototype relation for accelerated testing, it is assumed that the environmental stresses are negligible relative to the internal stresses of the system. In the remainder of this report, a more general relation is developed in which the effects of the environmental and internal stresses are both included.

Decomposition of the Generalized Stress

Suppose that the generalized stress is a function of generalized time, so that $S = S(T)$. Further, suppose the generalized stress may be decomposed into a mean, or nonperiodic, component, $S_M(T)$, and a cyclic, or periodic, component, $S_A(T)$, so that:

$$S(T) = S_M(T) + S_A(T),$$

where it is assumed that the component stresses are additive. If the generalized mean stress is very small relative to the periodic stress, then it may be neglected. This is shown by writing the preceding equation in the form:

$$S(T) = S_A(T)\left[1 + \frac{S_M(T)}{S_A(T)}\right],$$

where

$$\frac{S_M(T)}{S_A(T)} << 1$$

implies that:

$$S(T) \approx S_A(T).$$

Similarly, if the periodic stresses are small relative to the nonperiodic stresses, it is found that:

$$S(T) \approx S_M(T).$$

For intermediate cases, the ratio of the magnitude of the periodic to the nonperiodic stress is useful in characterizing the composition of the total stress. Thus, let:

$$\lambda = \frac{S_A(T)}{S_M(T)},$$

and note that the total stress is characterized as nonperiodic or periodic, according to whether λ is zero or infinite.

The Stress-Range Diagram

The preceding characterization of the total generalized stress is analogous to the representation of combined periodic and nonperiodic stresses imposed on a specimen in metal-fatigue studies. Figure 13 is a sketch of an illustrative "stress-range diagram" used in such studies.

For fatigue studies, S_M represents the mean stress applied to the specimen, and S_A represents the amplitude of the alternating stress applied to the specimen. The axes are scaled linearly in the plot and the rays emanating from the origin have slopes equal to λ. Thus, for $\lambda = 0$, the alternating stress is zero, and for $\lambda = \infty$, the mean stress is zero. For intermediate values of λ, the specimen is subjected to a combined mean and alternating stress. The curved contours represent constant times to failure, with increasing times to failure in those contours nearer the origin. The point designated by A on the plot shows that a specimen subjected to a combined mean stress of 50×10^3 lb/in.^{2} and an alternating stress of 10×10^3 lb/in.^{2} is expected to fail after 10 hours of testing. The value of λ for this case is 0.20. For the same value of λ, that is, for the same ratio of alternating stress to mean stress, the time to failure is shown along the ray OA. The point B indicates that a specimen tested at the same

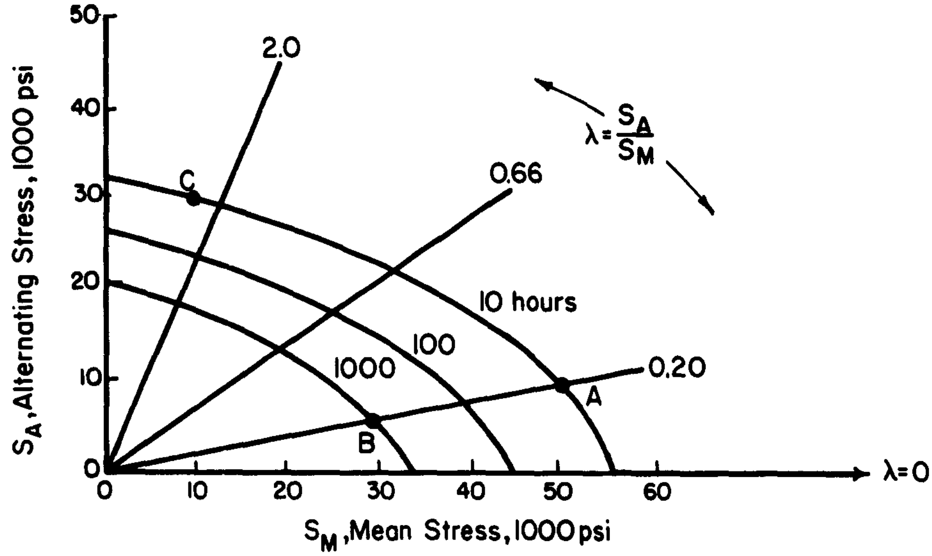

FIGURE 13. STRESS-RANGE DIAGRAM USED IN METAL-FATIGUE STUDIES

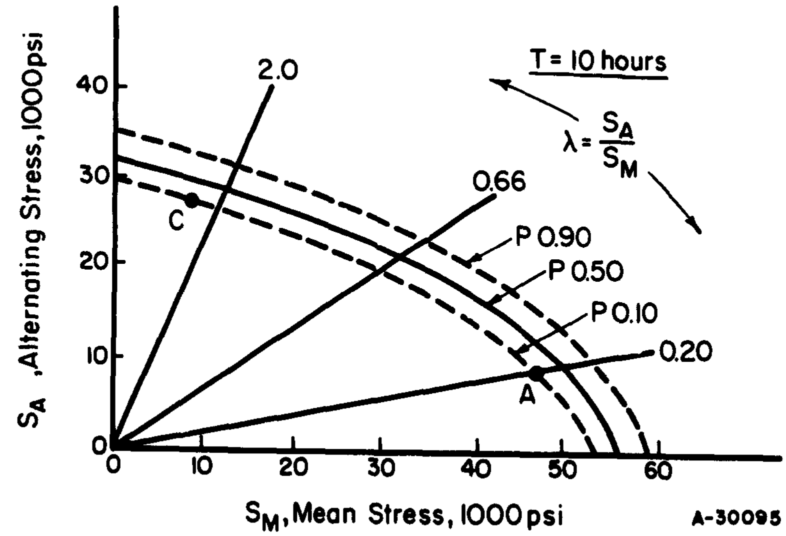

FIGURE 14. STRESS-RANGE DIAGRAM WITH FAILURE PERCENTILES

value of λ but at a lower mean stress, equal to 30×10^3 lb/in.^{2}, is expected to fail after 1000 hours. Along the contour lines, the expected time to failure is constant. These curves represent the "trade-off" curves between mean stress and alternating stress for a specified time to failure. The points A and C indicate that the same time to failure is expected for a combination of $S_M = 50 \times 10^3$ lb/in.^{2} and $S_A = 10 \times 10^3$ lb/in.^{2} as for the combination of $S_M = 10 \times 10^3$ lb/in.^{2} and $S_A = 30 \times 10^3$ lb/in.^{2}.

In this interpretation of the stress-range diagram, the time to failure is the expected, or arithmetic-mean, time to failure observed in testing a set of specimens at the test conditions. It is clear that a criterion other than the arithmetic mean may be used as a basis for the stress-range diagram. If sufficient data are available, the 0.10, 0.50, and 0.90 percentiles may be used, as shown for one contour in Figure 14. For this point, 10 per cent of the models tested at a generalized mean stress of 50×10^3 lb/in.^{2} and a generalized alternating stress of 7×10^3 lb/in.^{2} failed after a generalized time of T = 10 hours. An analogous statement is valid for the point C.

Finally, it is desirable to express the generalized stress-range diagram in a dimensionless form, as shown in Figure 15. The justification for the changes in symbolism shown in Figure 15 is obtained from the assumption that the probability of failure is expressible as a function of $\sigma = S/S_o$ and $\tau = T/T_o$; that is,

$$p = f(\sigma, \tau) = f(S/S_o, T/T_o).$$

This representation of p is generalized to indicate the decomposition of S into a periodic component, S_A, and a nonperiodic component, S_M, as shown by:

$$S = S_A + S_M = S_M \left[1 + \frac{S_A}{S_M}\right] = S_M (1 + \lambda).$$

Division by the characteristic stress, S_o, yields:

$$\frac{S}{S_o} = \frac{S_M}{S_o}(1 + \lambda),$$

where S_M/S_o and λ are mathematically independent. Thus, (S/S_o) in the functional representation of p must be replaced by two variables, (S_M/S_o) and λ:

$$p = f\left(\frac{S_M}{S_o}, \lambda, \frac{T}{T_o}\right).$$

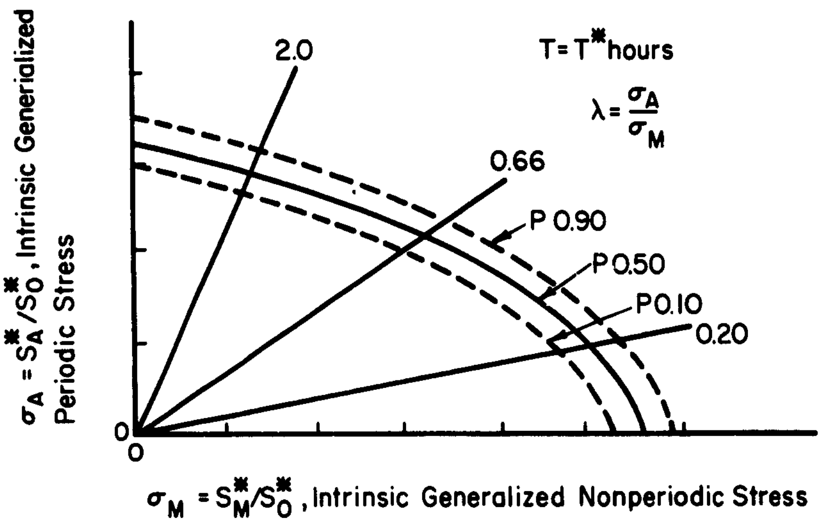

FIGURE 15. DIMENSIONLESS FORM OF STRESS-RANGE DIAGRAM

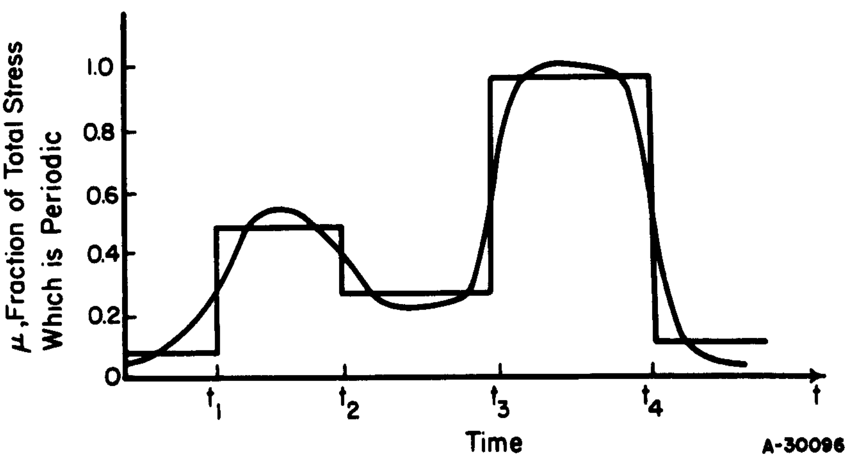

FIGURE 16. HYPOTHETICAL SKETCH OF THE PERIODIC PROPORTION OF TOTAL STRESS AS A FUNCTION OF TIME

Alternatively, the variables involved in the functional form of p may be written as follows:

$$p = g\left(\frac{S_M}{S_o}, \frac{S_A}{S_o}, \frac{T}{T_o}\right).$$

Because the ratio of (S_A/S_o) to (S_M/S_o) is equal to λ, the second form is clearly equivalent to the first form. Although both of these forms appear to have certain advantages, the second form will be used in the following discussions. By defining the intrinsic generalized periodic stress by $\sigma_A = S_A/S_o$ and the intrinsic generalized nonperiodic stress by $\sigma_M = S_M/S_o$, the form for p becomes:

$$p = f(\sigma_M, \sigma_A, \tau).$$

The advantages of the generalized stress-range diagram are made evident by noting that the model-prototype relations become:

(1) $\sigma_M = \sigma'_M$,

(2) $\sigma_A = \sigma'_A$,

(3) $\tau = \tau'$,

(4) $f = f'$,

(5) $S_o = (S_o)'$,

and

(6) $T_o \neq (T_o)'$.

The generalized stress-range diagram explicitly involves the quantities σ'_M, σ'_A, and τ', and these are numerically equal to the corresponding quantities for the prototype. Consequently, the contour lines, which are plotted according to the results of the tests on the models, may be read directly as applying to the prototype. This assumes, of course, that the functions f and f' have not changed in the transition between model and prototype. The model time associated with the plot is converted to prototype time by use of the time-scale factor.

Complications Involved in Time Dependence

The preceding discussion of the generalized stress-range diagram is based on the assumption that λ is not a function of time. That is, it is assumed that the ratio of the generalized periodic stress to the generalized nonperiodic stress is constant during the life of the system. For a system that is sometimes in storage, sometimes in a state of "mild" operation, and sometimes in a state of "severe" operation, it is clear that the assumption of a constant ratio of σ_A/σ_M over time is not valid. To account for various states of operation, the explicit dependence of λ on time is required. For many systems, such a dependence may be extremely difficult to formulate.

Alternative Representation of Stress Decomposition

From the equations shown previously, the intrinsic generalized stress may be represented by:

$$\sigma = \sigma_M(1 + \lambda),$$

where λ ranges between 0 and ∞. The range of λ does not permit a convenient description of time dependence by graphical plots. Consequently, the factor λ is replaced by a factor μ that is more suitable for graphic representation.

From the relations:

$$S_A + S_M = S$$

and

$$S_A/S_M = \lambda,$$

it follows that:

$$S_A/S = \lambda/(1 + \lambda).$$

Thus, the ratio $\lambda/(1 + \lambda)$ represents the fraction of the total generalized stress that is periodic. This ratio is represented by μ, so that:

$$\mu = \lambda/(1 + \lambda).$$

It is clear that, as λ ranges between 0 and ∞, μ ranges between 0 and 1.

With this definition of μ, the time dependence of λ induces a time dependence of μ. For illustrative purposes, it is supposed that Figure 16 represents a plot of μ as a function of actual time. The sketch indicates that the system is first subjected to a stress that is approximately 10 per cent periodic. At time t_1, the system is subjected to an operation having a 50 per cent periodic component. The segmented curve represents an approximation to the smooth curve under various time intervals. Figure 17 shows a plot of the intrinsic generalized periodic stress as a function of time. The smooth curve is again approximated by average values taken over the same time intervals as those used in the plot of μ.

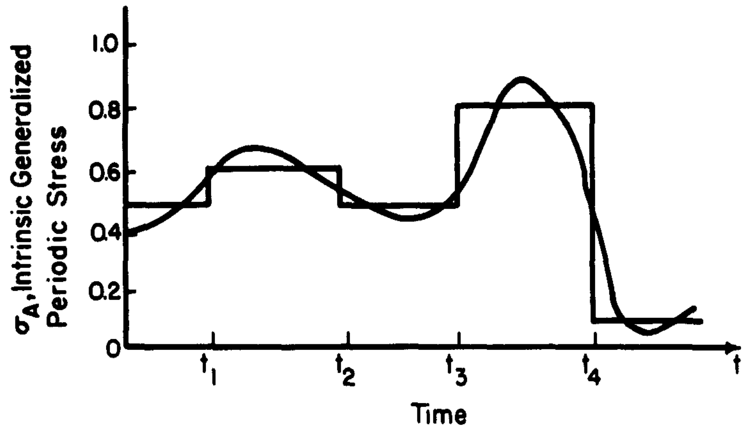

FIGURE 17. HYPOTHETICAL SKETCH OF INTRINSIC GENERALIZED PERIODIC STRESS AS A FUNCTION OF TIME

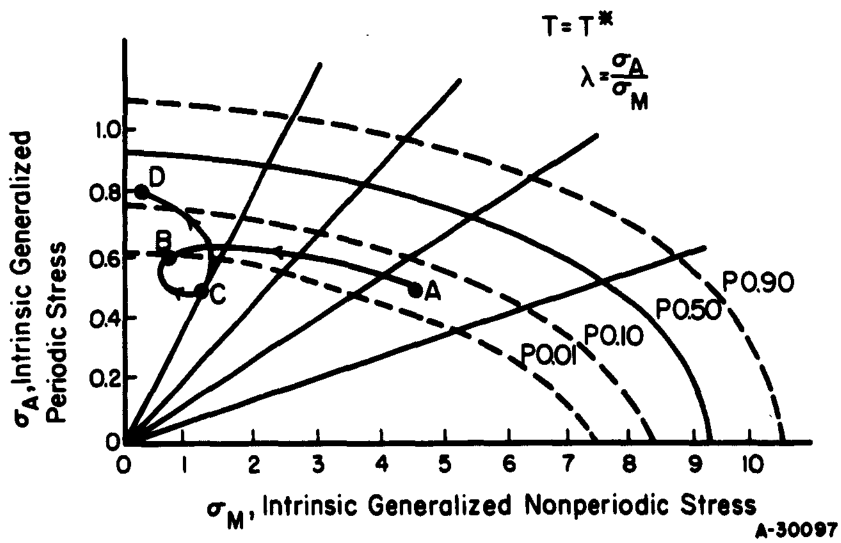

FIGURE 18. TIME TRAJECTORY OF A SYSTEM PLOTTED ON A GENERALIZED STRESS-RANGE DIAGRAM

The corresponding values of μ and σ_A obtained from the two preceding figures are next plotted on the generalized stress-range diagram. The result is a time trajectory on the stress-range diagram, which is suggested by Figure 18, where it has been assumed that real time and generalized time are identical for this example. As indicated on the figure, the system begins its trajectory at point A and traverses the path to the point D. The computation of the first point of the trajectory is outlined as follows: For the first time interval, the two preceding sketches indicate that $\mu = 0.1$ and $\sigma_A = 0.5$. From $\lambda = \mu/(1 - \mu)$, it follows that $\lambda = 1/9$, and from $\sigma_M = \sigma_A/\lambda$, it follows that $\sigma_M = 4.5$. The point $(\sigma_A, \sigma_M) = (0.5, 4.5)$ is shown as A in Figure 18. The successive points B, C, and D were obtained in a similar manner and were then joined by the smooth curve shown in the sketch. As indicated in the sketch, the point D shows the trajectory has crossed the contour representing a probability of failure of more than 10 per cent. If this trajectory were based on averages obtained from model testing, then the conclusion would also hold for the prototype; that is, at point D, the prototype has a failure probability that exceeds 10 per cent. The prototype time associated with this probability is obtained from the model time by using the time-scale factor.

Interpretation of Trajectory on Stress-Range Diagram

To clarify the meaning of the time-trajectory plot on the stress-range diagram, it is convenient to consider a special case. Suppose μ and σ_A are both constant over time, so that plots of these quantities appear as shown in Figure 19. From the plot of μ, it is seen that $\mu = 0.10$, so that 10 per cent of the total generalized stress is periodic. From the relation:

$$\sigma_M = \left(\frac{1 - \mu}{\mu}\right) \sigma_A,$$

it follows that:

$$\sigma_M = 9/2 = 4.5.$$

Thus, the point with coordinates $(\sigma_M, \sigma_A) = (4.5, 0.50)$ represents a component subjected to a total generalized stress, σ_T, given by:

$$\sigma_T = \sigma_M + \sigma_A = 4.5 + 0.5 = 5.0,$$

of which 10 per cent is a periodic component. This point is shown in Figure 18 as point A, and by extrapolation of the values shown on the stress-range contours, it is seen that the probability that the component will fail within T* hours is equal to 0.005, approximately. In this special case, the entire "trajectory" is concentrated at the point A, so that the point does not describe a path. It is under this condition that the stress-range diagram

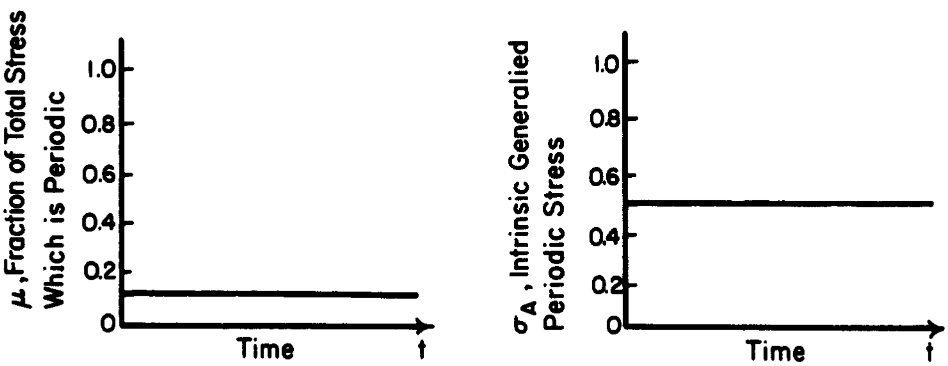

FIGURE 19. HYPOTHETICAL PLOTS OF SYSTEM STRESSES AS A FUNCTION OF TIME

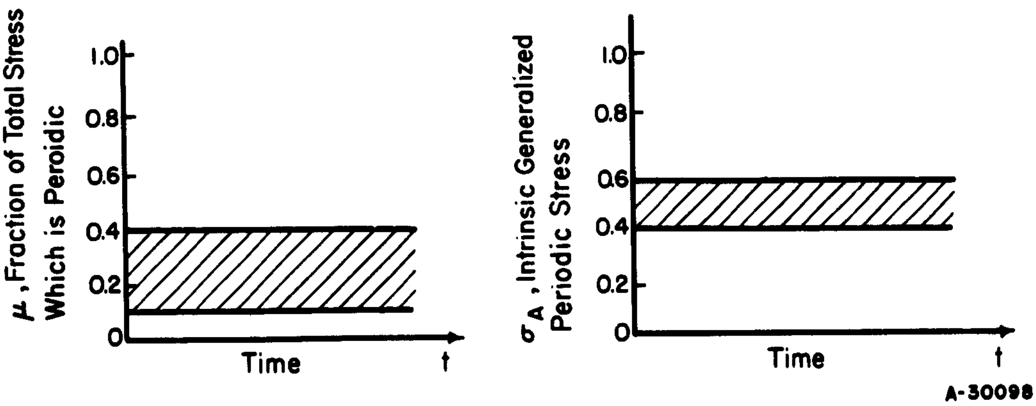

FIGURE 20. HYPOTHETICAL BOUNDS ON SYSTEM STRESSES AS A FUNCTION OF TIME

customarily applies. The extension of the diagram to include a point that moves with time is not ordinarily considered in metal-fatigue studies.

As a second special case, consider a component that remains at the point A for a number of hours equal to 0.75 T*, and for the remaining 0.25 T* hours assumes stress conditions that characterize the point B in Figure 18; namely, $\sigma_M = 3.0$ and $\sigma_A = 0.5$. If the path between A and B is considered to be described in an infinitesimally short period of time, then, for practical purposes, it is permissible to consider the component to be located at A for 0.75 T* hours and then located at B for 0.25 T* hours. It is desired to determine the probability that the component will fail within T* hours. From Figure 18, it is seen that:

(1) If the component stayed at A for T* hours, then the probability of failure within T* hours would be equal to 0.05, approximately, and

(2) If the component stayed at B for T* hours, then the probability of failure within T* hours would be equal to 0.01, approximately.

From these observations, one method for determining the probability of failure associated with a component at A 75 per cent of the time and at B 25 per cent of the time is given by $\overline{p} = (0.75)(0.05) + (0.25)(0.01) = 0.04$. That is, the probabilities at A and B are weighted in proportion to the time spent at A and B, respectively. More generally, if a component may be regarded as located at point A for (t_A/T^*) hours and at point B for (t_B/T^*) hours, where $t_A + t_B = T^*$, then an average probability to associate with the trajectory consisting of the points A and B is given by $\overline{p} = (t_A/T^*)\, p_A + (t_B/T^*)\, p_B = (t_A p_A + t_B p_B)/T^*$.

If the trajectory is composed of a sequence of M discrete points, then the average probability of failure may be represented by:

$$\overline{p} = \frac{1}{T^*} \sum_{i=1}^{M} p_i \, \Delta t_i, \; i = 1, \ldots, M,$$

where

$$\sum_{i=1}^{M} \Delta t_i = T^*,$$

and $\Delta t_i/T^*$ denotes the proportion of the total time spent by the system under stress conditions having an associated probability of failure given by p_i. Generalizing still further, if the number of points becomes arbitrarily large, so that $n \rightarrow \infty$, then the probability of failure is given by:

$$\overline{p} = \lim_{M \to \infty} \frac{1}{T^*} \sum_{i=1}^{M} p_i \, \Delta t_i = \frac{1}{T^*} \int_0^{T^*} p(t)dt.$$

If the trajectory is represented by a smooth curve on the generalized stress-range diagram, then the above equation associates a probability with the trajectory. In general, this equation indicates that the average probability of failure varies with the path taken between two points, A and B.

Difficulties in Interpretation of Trajectory Diagram

In the preceding development, several important assumptions are made. For example, it is assumed that the stress-range contours do not change with time. That is, the stress-range contours are assumed to form an invariant grid within which the failure probabilities may be read by interpolation. This assumption needs careful consideration, as evidenced by the fact that at point B of Figure 18, for example, the system has undergone a stress environment represented by the path from A to B. The system at B has been "prestressed", and, consequently, its failure probability may be different from that of a system that has not been prestressed. The difference is more clearly brought out by considering a system, K, that remains at point A for 75 per cent of the total time, and then undergoes the stress environment associated with point B for the remaining 25 per cent of the time. The "state" of such a component may be contrasted with the "state" of a second component, K', that remains throughout the time interval at conditions associated with point B. The difference in state between K and K' after 75 per cent of the total time has elapsed is this: System K has been subjected to a stress environment represented by point A, whereas system K' has been subjected to a stress environment represented by point B. Both of these environments may be regarded as "prestressing" stages for a time interval consisting of the last 25 per cent of the total time interval. The difference between prestressing at point A and prestressing at point B may be reflected as a difference in the probabilities of failure during the last 25 per cent of the time interval. When using the method proposed above, it is assumed that no change in failure probabilities occurs, so that, basically, it is assumed that prestressing at point A is equivalent to prestressing at point B. Such an assumption would appear to be unwarranted except for those trajectories that coincide with the contours of the stress-range diagram. To be conservative, it would appear necessary to restrict attention to those trajectories on the stress-range diagram that are nearly coincident with the stress-range contours.

In a practical application, the contours of the stress-range diagram will not be represented by simple contours, but, rather, by "confidence bands". The widths of the confidence bands will depend on the variability of the test results and on the number of models tested. Thus, in practice, it would appear that attention should be restricted to those trajectories that fall within a confidence band associated with a stress-range contour.

A second assumption involved in the preceding development is this: It is assumed that the probability of failure associated with a trajectory directed from A to B is the same as that for a trajectory directed from B to A, whenever corresponding segments of the path are traversed in equal time intervals. As an illustration, it is assumed that prestressing at a point A for 75 per cent of the total time followed by the environmental stress condition at point B for 25 per cent of the time yields the same failure probability as prestressing at B for 25 per cent of the time followed by the environmental stress at point A for the remaining 75 per cent of the time. It would appear difficult to assess the validity of this assumption without empirical data.

Use of Practical Bounds to Determine Limits on Trajectory Diagrams

As a final generalization, suppose that the plots of μ and σ_A appear as shown in Figure 20. The diagram on the left indicates that the proportion of the total stress that is periodic varies between 10 and 40 per cent; the diagram on the right indicates that the generalized alternating stress varies between 40 and 60 per cent of the total generalized stress. By combining the upper and lower values of μ with each value of σ_A, the coordinates of the points A, B, C, D on the stress-range diagram may be obtained. For the numerical values shown on Figure 20, it follows that the coordinates of these points are given by the table:

P:	(σ_M, σ_A)
A:	(5.4, 0.6)
B:	(3.6, 0.4)
C:	(0.9, 0.6)
D:	(0.6, 0.4)

These points are shown on the trajectory diagram in Figure 21.

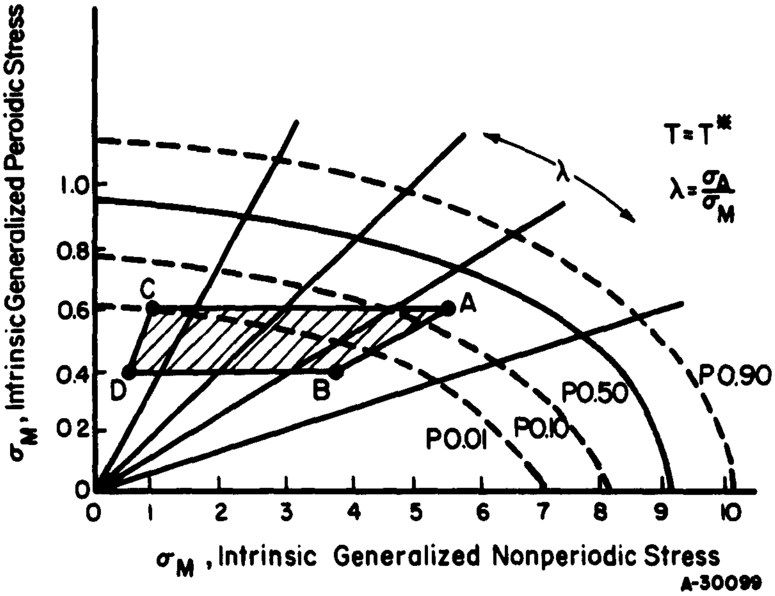

FIGURE 21. BOUNDS FOR TIME TRAJECTORIES OF A SYSTEM WITH STRESSES SHOWN IN FIGURE 20

The shaded portion of Figure 21 represents those points of the stress-range diagram that are consistent with the bounds imposed by the plots of μ and σ_A. It is clear that the procedure may be reversed, so that, starting with a shaded region of the stress-range diagram, the corresponding bounds on μ and σ_A may be determined. This procedure may represent a useful application of the stress-range diagram. A shaded region may be selected that has associated with it failure probabilities that are sufficiently small for the application involved. The corresponding σ_A and μ will then furnish the limits for the generalized periodic and nonperiodic stresses under which the component should be operated.

Other applications of this formulation are suggested by considering various functional forms for the generalized periodic and nonperiodic stresses. For example, what combinations of linearly increasing mean stress and exponentially decreasing periodic stress yield the largest expected life? For a prescribed functional form of the generalized mean stress, what should be the form of the generalized alternating stress that yields a trajectory having a constant failure probability? Specific examples of this type can be easily examined by using the preceding methods whenever a practical situation requires it.

CONCLUSIONS

Basic concepts usually associated with the theory of models can be applied to the prediction of catastrophic failures of systems that are highly reliable. Paperwork computations of the reliability of such systems are often unconvincing when made without benefit of actual data. Moreover, because excessively long periods of actual operation are required to obtain a single catastrophic failure, the reliability of such a system is often beyond direct experimental inquiry. As an alternative to both of these procedures, the theory of models furnishes criteria and methods suitable for the rapid generation of experimental data that may be used to predict the reliability of reliable systems. Mathematical criteria developed in this report indicate the conditions under which reliability data obtained from "accelerated" tests can be used to predict reliability under "normal" use.

The simple model-prototype relation developed is suitable for application to accelerated test programs when environmental stresses are negligible. The proposed experimental procedure involves a set of "accelerated models" that are tested under high "abuse ratios" until one catastrophic failure occurs. The equations and methods required to use this information to predict the probability of failure of the prototype, which operates under normal use, make use of both nonparametric statistical methods and the assumption of an exponential distribution of failure times. The equations derived may be used to determine the maximum possible "acceleration" for the models and the probability of failure under normal use for various operating times at various confidence levels.

Because the primary concern in the prediction of catastrophic failures is with the size of the "tail" that contains early failures, it is concluded that "mean life" requirements are not appropriately applied to the catastrophic reliability of highly reliable systems. Instead, it is proposed that a catastrophic reliability requirement should take a form similar to the following: "It must be demonstrated with 90 per cent confidence that the probability is 0.05 that the system will suffer a catastrophic failure within 10^6 operating hours under normal use." The methods derived yield conclusions of this form even though the distribution of failure times and the associated mean life are unknown.

The general model-prototype relation that is presented considers the case in which the total generalized stress on the system consists of a periodic and a nonperiodic component. The graphical method of analysis proposed is analogous to the "stress-range diagram" usually associated with metal-fatigue studies. The principal use of this method may be the determination of bounds within which a system is required to operate in order to meet specific reliability requirements.

Finally, it is concluded that the theory of models furnishes a highly flexible and singularly appropriate framework for the development of quantitative criteria and methods suitable for a wide variety of reliability problems. It is believed that the results obtained in this report demonstrate that continued efforts involving model-theoretic approaches to reliability are justified.

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APPENDIX D

FIELD-DATA-COLLECTION PROGRAM

APPENDIX D

FIELD-DATA-COLLECTION PROGRAM

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APPENDIX D

FIELD-DATA-COLLECTION PROGRAM

Effective use of the quantitative statements of reliability requires reasonably accurate data on a large number of variables present in the operational environment. What is needed is a continuous history of the use, failure, and repair of a defined set of propeller controls in the context of the actual operational environment. Existing methods for collecting these types of data on operations and maintenance in the USAF provide information primarily for control of operations and maintenance and for estimating the kind and number of replacement parts needed in the inventory. As a part of the research under this contract, a field-data-collection program was designed and implemented to provide as much information as possible on the operational environment of turboprop propeller controls in service.

Methods and products of data-collection processes now existing in the USAF were examined to determine their usefulness for estimating reliability. This was done through discussions with key personnel at all levels of USAF maintenance activity. It was concluded that certain essential data required to define reliability relations were not collected in the normal routine. These include precise identification and operating history of a failed part, the population of such parts operating in a given time, and the estimated cause and observed effect of any given malfunction. Because it is desirable to minimize interference with routine operations, the data-collection plan was designed to use existing procedures wherever possible with only minor modifications, and to minimize the effort of USAF personnel.

Data-Collection Plan

Only limited numbers of turboprop aircraft have reached operational status in the USAF, primarily in the support wings of the Tactical Air Command. This fact limited the choice and location of equipment to be studied. The Operational Wing selected for study possessed 51 turboprop aircraft, equally distributed among three squadrons. In August, 1957, the Wing was preparing to convert its organization to the consolidated maintenance management operation set forth in TACM 66-1, dated July, 1957. This form of maintenance organization simplifies the process of collecting failure data at a central point. Since there were a large number of low-time aircraft, a data-collection program for this Wing offered an opportunity to monitor reliability growth or deterioration throughout most of the contemporary life history of the propeller controls. Personnel in the operating units were most helpful in the decision to establish the data-collection program at this Base.

Reporting Media

Organization of a data-collection program depends on what data are to be collected and how and in what form they are to be reported. Examination of the existing information channels revealed that DD Form 781-2, Aircraft Inspection and Maintenance Record, could provide valuable information with only slight modification in reporting procedure. This daily record indicates aircraft status, flying hours, and most maintenance actions taken on a specific aircraft. A simple modification in the reporting process was made by requesting the propeller specialist to make a separate entry for each propeller maintenance action accomplished. Each propeller specialist was provided with a set of rubber stamps detailing the information desired. The use of these rubber stamps would standardize the data entry and minimize the writing required. An instruction sheet, shown in Figure 22 was given to each propeller specialist as a guide in reporting each propeller maintenance action accomplished.

The DD Form 781-2 contains a record of most maintenance actions performed during the normal working period. As a supplement to this, it was necessary to review the AFTO Form 26's that are used to record maintenance actions during periodic inspections.

In routine operational flying, detailed descriptions of the flight operating conditions at the time a malfunction occurs are seldom recorded in permanent form. Knowledge of flight conditions and operating conditions at the time of malfunction could have a strong bearing on reliability estimates. The effect of the malfunction on the performance of the assigned mission is particularly important in evaluating the effect of reliability on operations. In the absence of standard procedures for collecting such information, a special form, Aircraft Propeller Malfunction Record, shown in Figure 23, was prepared for use in this program. The Aircraft Propeller Malfunction Record is not a standard USAF form. For this reason, it was necessary to obtain specific approval for its use from the Air Material Command. Instructions for completing the special form were provided as shown in Figure 24. Flight technicians or flight engineers were instructed to complete this form for each propeller malfunction experienced whenever the propeller was operating in flight or on the ground. This record could then be correlated with subsequent maintenance actions indicated on the DD Form 781-2.

In addition to the data discussed above, several corollary items of information were collected. A monthly summary was made showing the total propeller maintenance work load in the consolidated Wing maintenance organization and a subtotal showing the maintenance work load on the various maintenance actions performed at the Base level. A study was also made of the propeller-control overhaul procedure at the cognizant Depot.

FIGURE 22. INSTRUCTIONS TO THE PROPELLER SPECIALIST FOR COMPLETING DD FORM 781-2

The Propeller Specialists are requested to make several additional entries on DD Form 781-2. These entries and the desired method for recording them are described below.

The additional entries on DD Form 781-2 are designed to obtain data needed by the Air Force about the operational environment and causes of malfunction of propeller controls in turboprop aircraft. This is an important part of a research program for the Propulsion Laboratory, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio

The objective of this special data-collection program is to provide information about propeller malfunctions and the environment in which they occur. The ultimate aim of the research program is to provide guidance for future efforts to attain and maintain propeller control reliability. Everyone working with turboprop aircraft and propeller controls will realize some benefit if it is possible to reduce accidents and incidents involving propeller control malfunctions and reduce the maintenance workload caused by an excessive number of malfunctions.

The information requested on DD Form 781-2 is important in obtaining a chronological record of propeller malfunctions, the maintenance action required, and an estimated cause of malfunction. Because such information is not now recorded or is not available in readily useful form, the Propeller Specialists are requested to make these entries in the DD Form 781-2 as indicated below.

REPORTING PROCEDURE

The following procedure should be followed for recording the additional information desired on DD Form 781-2: For each maintenance action on a propeller including the propeller controls, an entry should be made in Column 26, Pilot's and Mechanic's Remarks, and Column 27, Corrective Action, providing the following information:

24. System	25. Sym	26. Pilot's and Mechanic's Remarks	27. Corrective Action	27 A. Time 66
12		*2* (Eng. No.) *Improper Propeller Speed* (Malfunction Reported or Observed) during *Preflight* (First Indication) Was due to *Cracked Compression Spring* (Cause of Trouble) on *Governor Mod. A* *41357* (Assy. Name, Model, Serial No.) *Compression Spring 1013* (Part Name, Model, Serial No.)	*Replaced Part* (Corrective Action Taken) *Compression Spring 1013* (Name, Model, Serial No. of Replacement) during *Unscheduled Maintenance* (Time of Action)	

Column 26

1. Engine Number in which malfunction occurred.
2. Description of the malfunction, i. e., what was reported or observed.
3. First indication of failure or malfunction (i. e., preflight inspection, postflight inspection, scheduled maintenance, unscheduled maintenance, or in-flight).
4. Description of part or component judged to be the probable cause of failure or malfunction.
5. Assembly Name, Model, and Serial Number, (i. e., governor, feathering dome, regulator, etc.).
6. Part or Component Name, Model, and Serial Number.

Column 27

1. Corrective action taken such as replacement, repair, adjustment, etc.
2. Name, Model, and Serial Number of replacement (if applicable).
3. Time of or reason for replacement, repair, or adjustment (T.O.C., scheduled maintenance, unscheduled maintenance, overhaul, etc.).

AIRCRAFT PROPELLER MALFUNCTION RECORD

1. Date	2. Aircraft Type Aircraft Serial No.	3. Mission Symbol Assigned This Flight (Form 781-7)	4. Effect of Malfunction on Assigned Mission (Check Items Pertinent) ☐ No effect ☐ Reduced performance ☐ Feathering ☐ Abort ☐ Emergency landing

GENERAL FLIGHT CONDITIONS AT TIME OF MALFUNCTION

5. Flight Phase When Malfunction Occurred (Check One) ☐ Ground ☐ Climb ☐ Landing ☐ Take-off ☐ Cruise	6. Altitude ______ Indicated Air Speed ______	7. Free Air Temperature ______ Other (Specify) ______	8. Aircraft Attitude (Check One) ☐ Straight and level ☐ Turn ☐ Climb ☐ Descent

PROPELLER AND ENGINE CONDITIONS AT TIME OF MALFUNCTION

9.

Turbine Inlet Temperature, F	1. ______	2. ______	3. ______	4. ______
Propeller, rpm	1. ______	2. ______	3. ______	4. ______
Engine Torque, in-lb	1. ______	2. ______	3. ______	4. ______

PROPELLER FEATHERING TIME PRIOR TO MALFUNCTION

10. Engine No. 1. ______ 2. ______ 3. ______ 4. ______

BRIEF WORD DESCRIPTION OF MALFUNCTION AND CORRECTIVE ACTION TAKEN BY AIR CREW

11. (Include any unusual operating conditions or indications of potential failure occurring before malfunction)

A-26733

FIGURE 23. AIRCRAFT PROPELLER MALFUNCTION RECORD

FIGURE 24. INSTRUCTIONS FOR COMPLETING SPECIAL FORM, AIRCRAFT PROPELLER MALFUNCTION RECORD

The Aircraft Propeller Malfunction Record is designed to obtain data needed by the Air Force about the operational environment of propeller controls in turboprop aircraft. This is an important part of a research program for the Propulsion Laboratory, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio.

The objective of this special data-collection program is to provide information about propeller malfunctions and the environment in which they occur. The ultimate aim of the research program is to provide guidance for future efforts to attain and maintain propeller control reliability. Everyone working with turboprop aircraft and propeller controls will realize some benefit if it is possible to reduce accidents and incidents involving propeller control malfunctions and reduce the maintenance work load caused by an excessive number of malfunctions.

The information requested on this special form is important in studying the environment when malfunctions occur. Because such information is rarely recorded in readily useful form, the Aircraft Propeller Malfunction Record is provided for this purpose.

REPORTING PROCEDURE

Entries on this form are self-explanatory, with the following exceptions:

Item 3 - Mission Symbol Assigned This Flight. Enter appropriate symbol designated on DD Form 781-7.

Item 6 - Altitude and Indicated Air Speed. Enter the approximate altitude to the nearest 1,000 feet observed at the time of the malfunction. Enter the approximate indicated air speed to the nearest 25 knots per hour observed at the time of the malfunction.

Item 7 - Free Air Temperature and Other. Enter the free air temperature to the nearest 10 degrees observed at the time of the malfunction. For "Other" enter any other climatic influences observed at the time of the malfunction such as heavy icing, severe winds, etc.

Item 9 - Turbine Inlet Temperature, Propeller RPM, and Engine Torque. Enter the data observed at the time of the malfunction in the spaces for appropriate engines and indicate the extreme values observed for the malfunctioning engine.

Item 10 - Propeller Feathering Time Prior to Malfunction. Enter, in the spaces for the appropriate engines, the number of times each propeller was feathered and the approximate total time feathered (in minutes) for this flight prior to the occurrence of the malfunction.

Data-Collection Process

The collection of data utilized a routine flow of information, as shown in Figure 25. The DD Forms 781-2 were sent daily from the Squadron maintenance officer to the Records Section. Because these forms must be on file in the Record Section for a 6-month period, it was necessary to reproduce the entire form in order to obtain the pertinent information for the data-collection program. Likewise, the AFTO Forms 26 containing propeller entries were reproduced for the record. Reproduction of these forms was accomplished by a Battelle representative during the semimonthly visits. Because the Aircraft Propeller Malfunction Record also originated at the Squadron level, the completed forms were attached to the DD Forms 781-2 and were collected in the Record Section.

The field-data-collection program was conducted during the period January to September, 1958. Additional data were obtained on malfunctions and operation of the turboprop aircraft prior to January, 1958, by reviewing the standard Air Force forms such as the DD Forms 781-2 and the AFTO Form 26. The pertinent forms of interest for the reliability study were available for the entire period beginning in May, 1957, when the turboprop aircraft were put in operation. Including the period prior to January, 1958, the data-collection program covered 688 aircraft months and a total of 86,000 propeller flying hours. Battelle personnel made frequent visits to the Base to monitor the program. During these visits, meetings were held with the Base personnel participating in the data collection to exchange information of interest and to encourage continuation of the always excellent cooperation. During the 9-month program, Battelle personnel made a total of 15 visits for periods of from 2 to 5 days each.

In a few cases, the pertinent Air Force forms were not available after June, 1958. This was because the aircraft were assigned temporarily to other bases, and the DD Forms 781-2 remain with the aircraft during the period away from the home Base. At the close of the field-data-collection program, approximately half of the original 51 aircraft included in the data-collection program were assigned to temporary duty. This is equivalent to a total of 54 aircraft months of operational data, and these data are not included in the analysis.

During the early phases of the field-data-collection program, an acceptable amount of data was obtained through the cooperation of the Base personnel, particularly on the special form, Aircraft Propeller Malfunction Record. At the end of May, 1958, however, operational conditions were affected by the world situation and a number of the aircraft were assigned to temporary duty off Base. The reporting of special data on operating conditions at the time of malfunction and special entries on DD Form 781-2 were essentially brought to a halt. Many of the Base personnel were placed

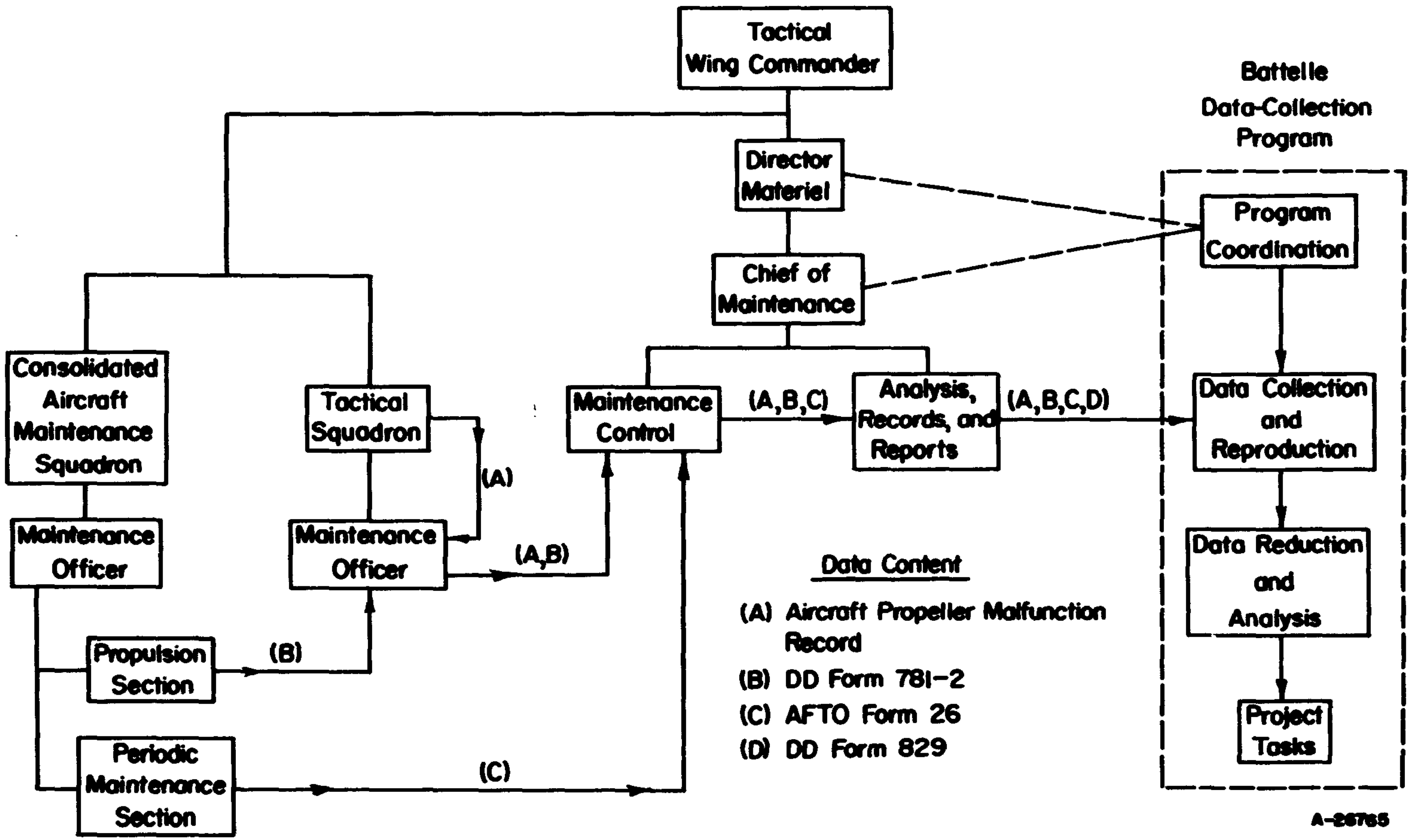

FIGURE 25. DATA COLLECTION PROCESS

on alert or were being transferred; those remaining were assigned extra duty. Efforts by the Battelle personnel to revive interest in the program and assure accurate reporting after June, 1958, were not successful.

Special Data Entries

One part of the special data collection was carried out by the propeller specialists. They were requested to make a special entry on the back of the DD Forms 781-2. As an aid, the propeller specialists were provided rubber stamps outlining the information desired and minimizing the writing required. The main purpose of the rubber-stamp entry was to provide information pertaining to the cause of a particular malfunction since this information is not available on the standard Air Force forms.

The propeller specialists at the Base level are more or less limited to adjustment, servicing, or the removal and replacement of the propeller assemblies, rather than repair of these assemblies. This limits their ability to find and report a cause for a particular malfunction. For example, propeller specialists are not allowed to dismantle a regulator, one of the major assemblies in the propeller-control system. Table 4 provides an indication of the limited work the propeller specialists perform at the Base level.

TABLE 4. TASKS AND JOB STANDARDS

(Prepared by Standardization Section)

Job Nomenclature	Remove and Rcplace	Adjust or Repair
(1) Strobe-run	N/A	3 man-hours (2 men)
(2) Gray box (circuit tester)	N/A	2 man-hours (2 men)
(3) Propeller	10 man-hours (2 men)	1 man-hour (1-2 men)
(4) Regulator	4 man-hours (2 men)	N/A
(5) Reservoir	1 man-hour (1 man)	N/A
(6) Spinner	10 man-hours (2 men)	2 man-hours (1 man)
(7) ATE	1 man-hour (1 man)	4 man-hours (1 man)
(8) Alternator	1 man-hour (1 man)	N/A

In many cases the normal entry made by the propeller specialists on the DD Form 781-2 contained all the information they could provide, particularly in cases where they were unable to give the estimated cause of the malfunction. In a few cases, the propeller specialists were able to give the true cause of a malfunction. These few data were not sufficient for analysis in any detail.

Aircraft Propeller Malfunction Record

The purpose of the Aircraft Propeller Malfunction Record was to obtain data on the flight and operating conditions at the time of a malfunction. This special form was particularly successful during the early phases of the data-collection program. In the period from January through May, 1958, an estimated one-third to one-half of the malfunctions were reported on the special form. After that period, only a few of the special forms were received each month. In general, the special forms that were received were completed satisfactorily. Because of the high inherent reliability of the propeller-control system and the difficulties encountered after May, 1958, the number of special forms received was not sufficient to allow an analysis to be made.

Depot Information

Knowledge of the Depot overhaul procedure would complete the history of the operational cycle of the propeller controls. It was concluded that this information would have to be taken from pertinent Technical Orders, Time Compliance Technical Orders, and Maintenance Engineering Orders covering overhaul of the propeller assemblies at the Depot. Plans were made to receive copies of these publications from the Depot at monthly intervals. It was learned late in the program that this information would not have been necessary, since only a few of the propeller assemblies had been sent to the Depot for overhaul. Most of the propellers included in the data-collection program had not yet reached the mandatory overhaul time of 600 hours. In October, 1957, this time was extended to 900 hours.

Results

The propeller and its control mechanism on the whole are exhibiting high reliability under the prevailing operating conditions. Two factors pertaining to propeller-control reliability are quite evident. First, the propeller-control malfunctions account for over 90 per cent of the total propeller malfunctions. The integrity of structural members, the blades, gears, shafts, etc., is good, with the result that the malfunctions that do occur are primarily associated with elements of the control mechanism, the "brains" rather than the "muscles". However, there is no certainty that recent design changes in the blade and gearing components to reduce rpm and cabin vibration will not influence the structural reliability. Further observation would be necessary to evaluate the effect of such changes on operational reliability. Secondly, the propeller repair workload is a rather small fraction of total aircraft maintenance workload. Considering the limited scope of the propeller repair activity shown in Table 4, a twofold increase in propeller malfunction rate or repair man-hours per malfunction would not significantly affect the operational reliability attainable.

Figures 26 through 35 contain a graphical summary of the operational data on the turboprop aircraft operation from the period May, 1957, to September, 1958. Included in this material are (1) the removal and replacement of the propeller assemblies and components, (2) malfunctions of the propeller assemblies and components, (3) the maintenance workload, and (4) the flying load. In studying these graphs, it should be remembered that the number of aircraft operating in the period June to September, 1958, was considerably smaller than the original population of 51 aircraft.

Figure 26 shows the premature removal rate per thousand propeller hours flown for propeller assemblies, regulators, ATE's, and alternators. Two other propeller assemblies, the spinner and the reservoir, that experienced premature removals during the data-collection program are not included because the total number removed was very small.

By using information obtained on the rubber-stamp entry on DD Forms 781-2 and also the Forms 26E, the history of the propeller assemblies after first removal was determined. Figure 27 shows that the lag time between removal and installation on another aircraft experienced by the majority of propeller assemblies was less than 2 to 3 weeks. This bears out the fact that maintenance action on these assemblies was done at the Base level, rather than the Depot level. Depot repair or overhaul of a propeller assembly would normally require a period of 6 to 8 weeks; however, only a few propeller assemblies had been sent to the Depot for overhaul. This verifies the lag-time chart on Figure 27.

Figure 28 indicates the distribution of the malfunctions experienced on the propeller. All control malfunctions were recorded under the propeller assembly, and also under the particular subassembly that malfunctioned. "Other Control", as used in the figure, includes the control malfunctions that were not attributed to a particular assembly. "Other", as used in the figure, includes propeller malfunctions not related to control.

Figure 29 shows the total propeller malfunctions per month and the propeller malfunctions per thousand propeller hours reported on the DD Forms 781-2. It is noted that the exceptionally high malfunction rate during October, 1957, and August, 1958, rises in the same manner as the premature removal rate for the ATE's, propeller assemblies, and regulators, respectively, as shown on Figure 26.

Figure 30 shows the total aircraft maintenance man-hours recorded on the DD Form 781-2 and also the total propeller maintenance man-hours expended per calendar month. The total number of man-hours expended on propellers was obtained through the cooperation of the Wing personnel in the Reports and Analysis Section. The man-hour figure includes all of the miscellaneous work done by the propeller personnel during their normal work day. An average of 85 per cent of the man-hours expended in the propeller section is carried out on miscellaneous work. The other 15 per cent is

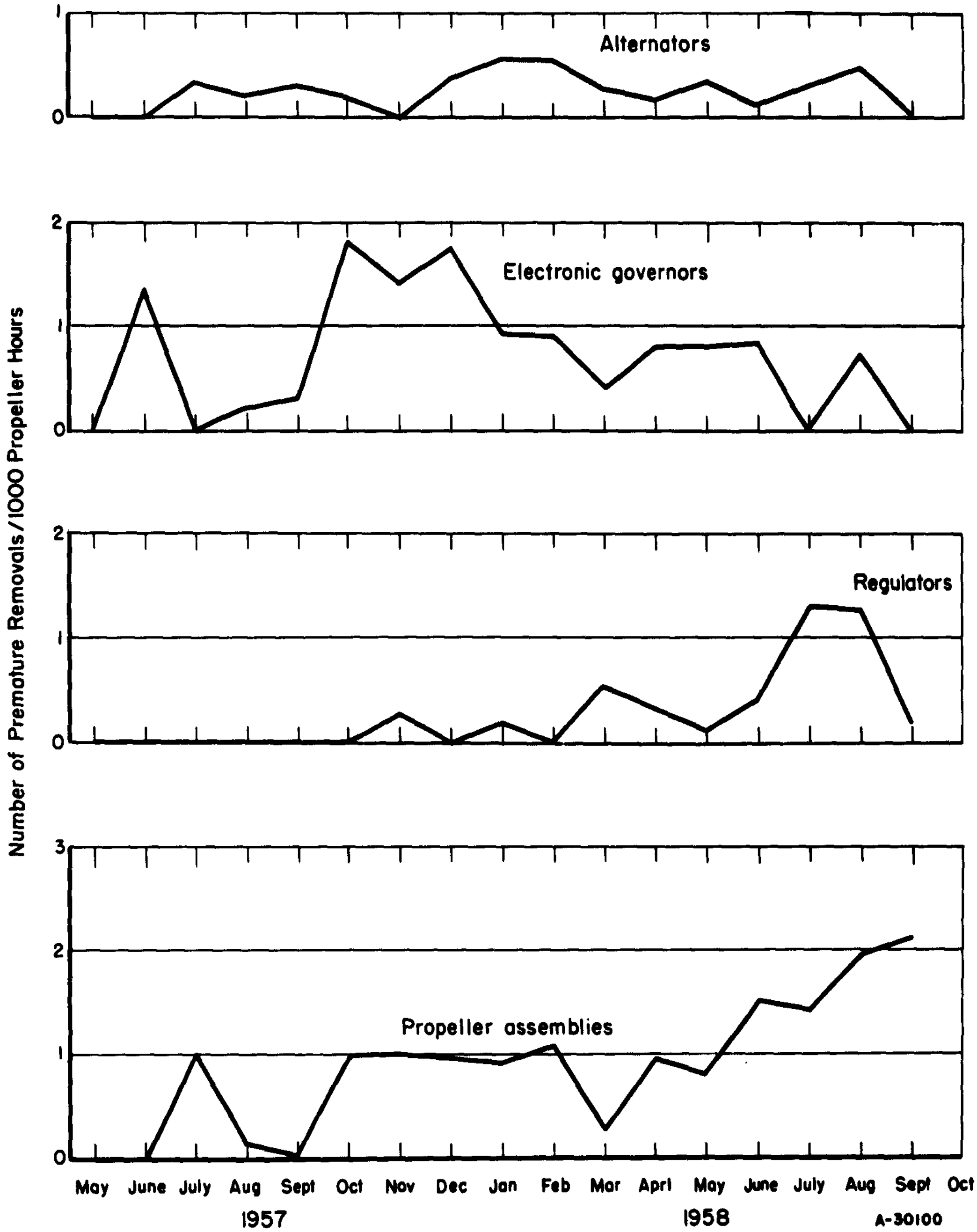

FIGURE 26. PREMATURE REMOVAL RATE FOR THE PROPELLER ASSEMBLY, THE REGULATOR ASSEMBLY, THE ELECTRONIC GOVERNOR (ATE), AND THE ALTERNATOR

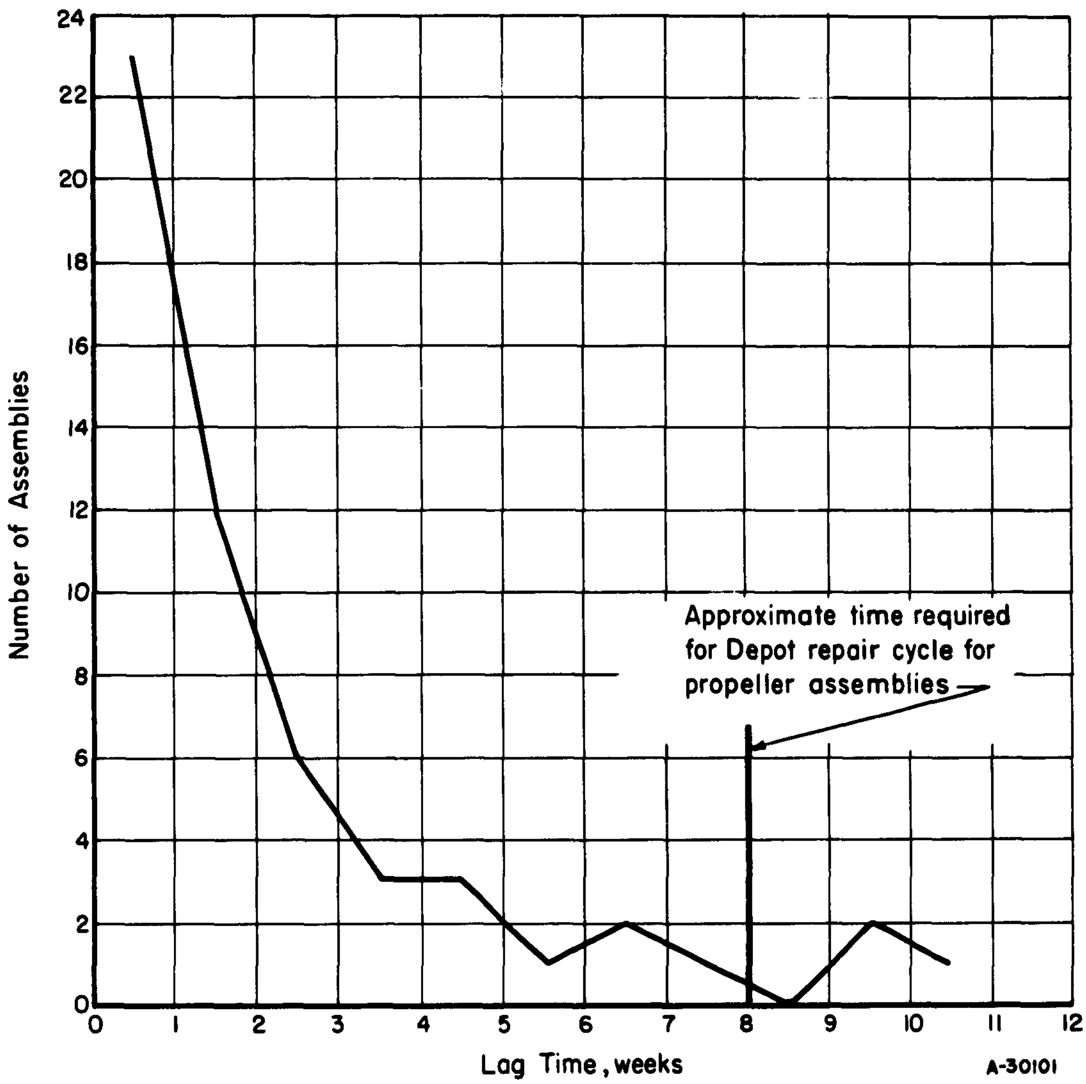

FIGURE 27. LAG TIME BETWEEN REMOVAL OF PROPELLER ASSEMBLY AND INSTALLATION ON ANOTHER AIRCRAFT

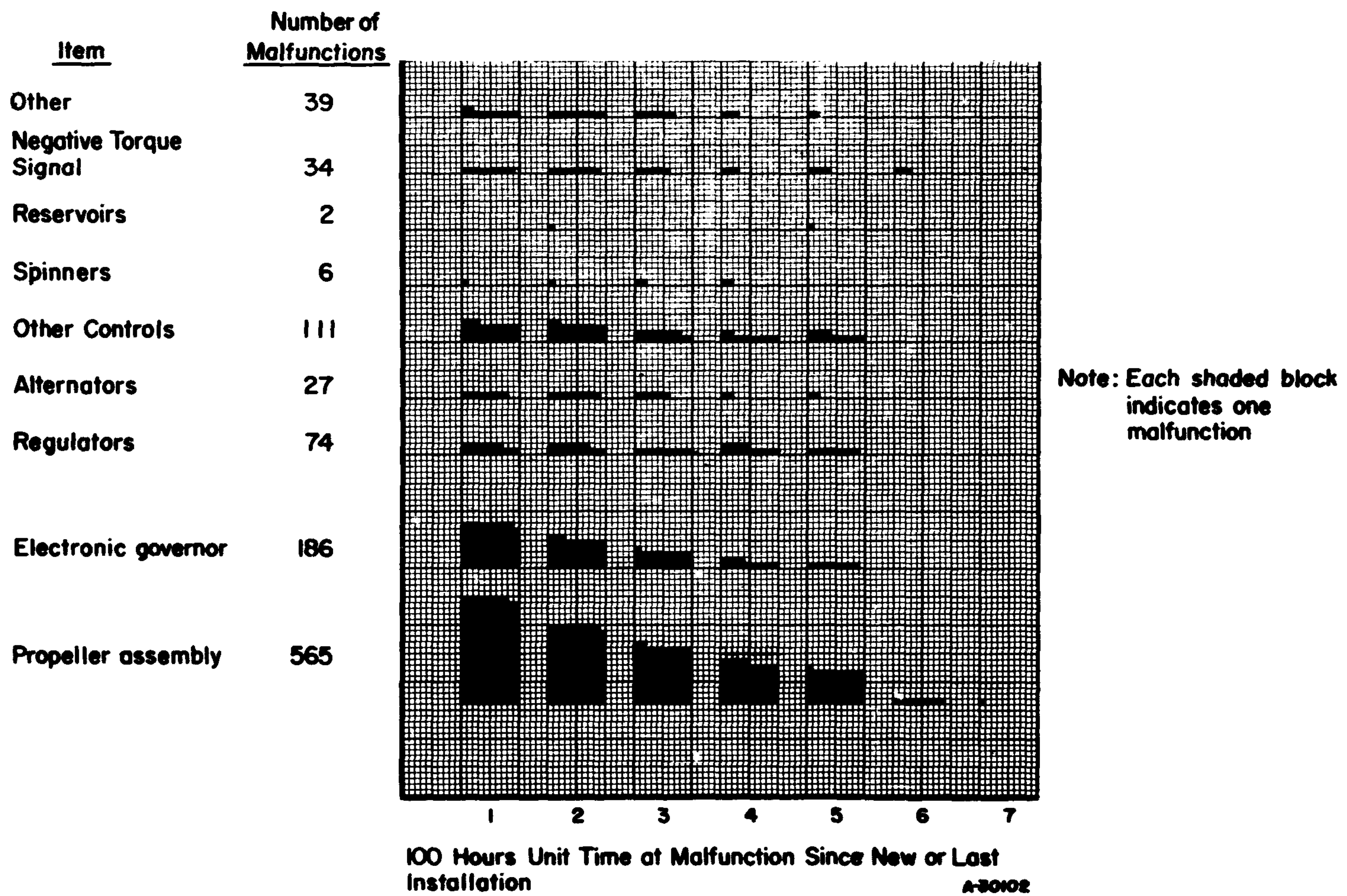

FIGURE 28. DISTRIBUTION OF THE PROPELLER MALFUNCTIONS AS RECORDED ON DD FORMS 781-2

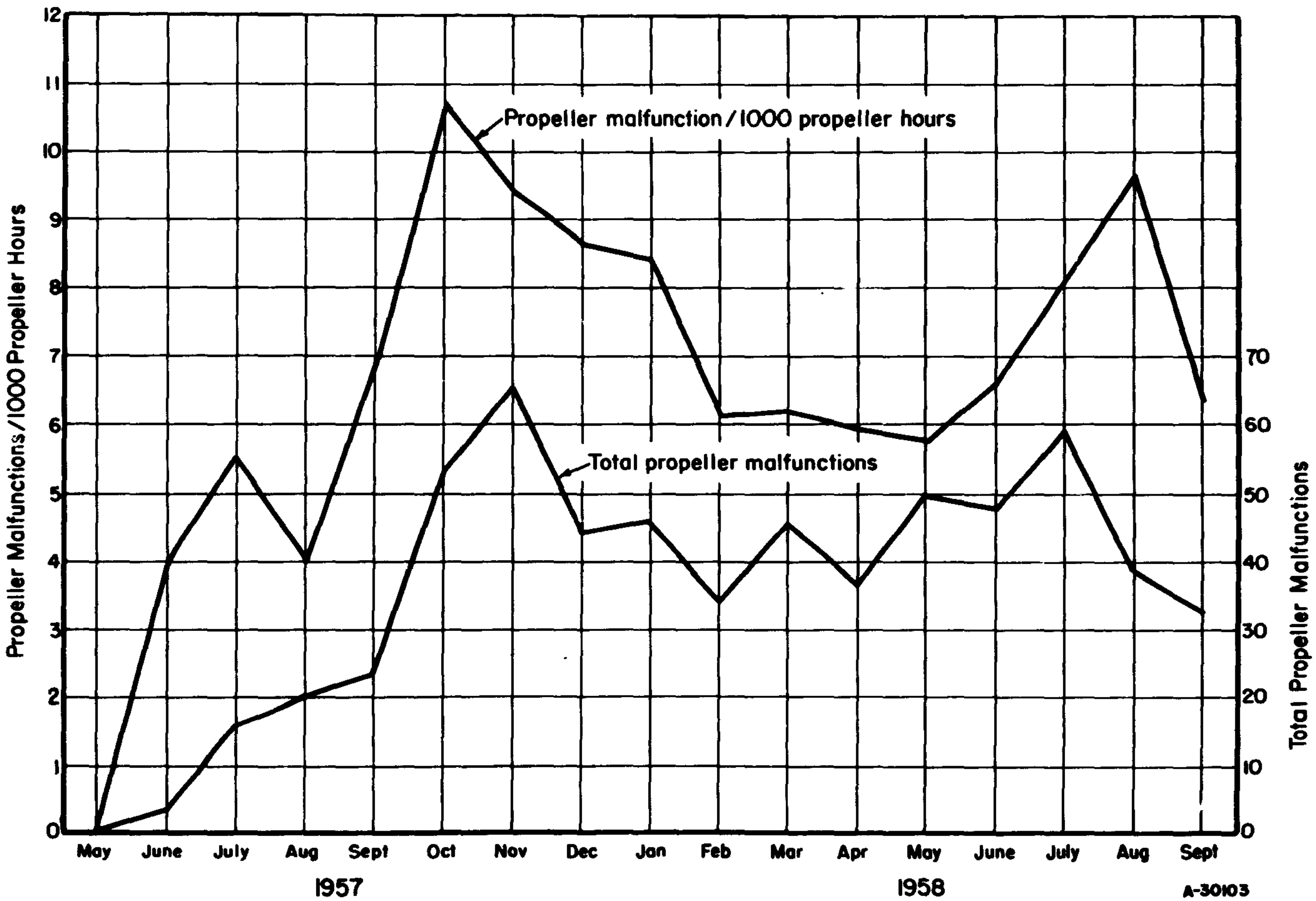

FIGURE 29. TOTAL PROPELLER MALFUNCTIONS AND PROPELLER MALFUNCTIONS PER 1000 PROPELLER HOURS AS REPORTED ON DD FORMS 781-2

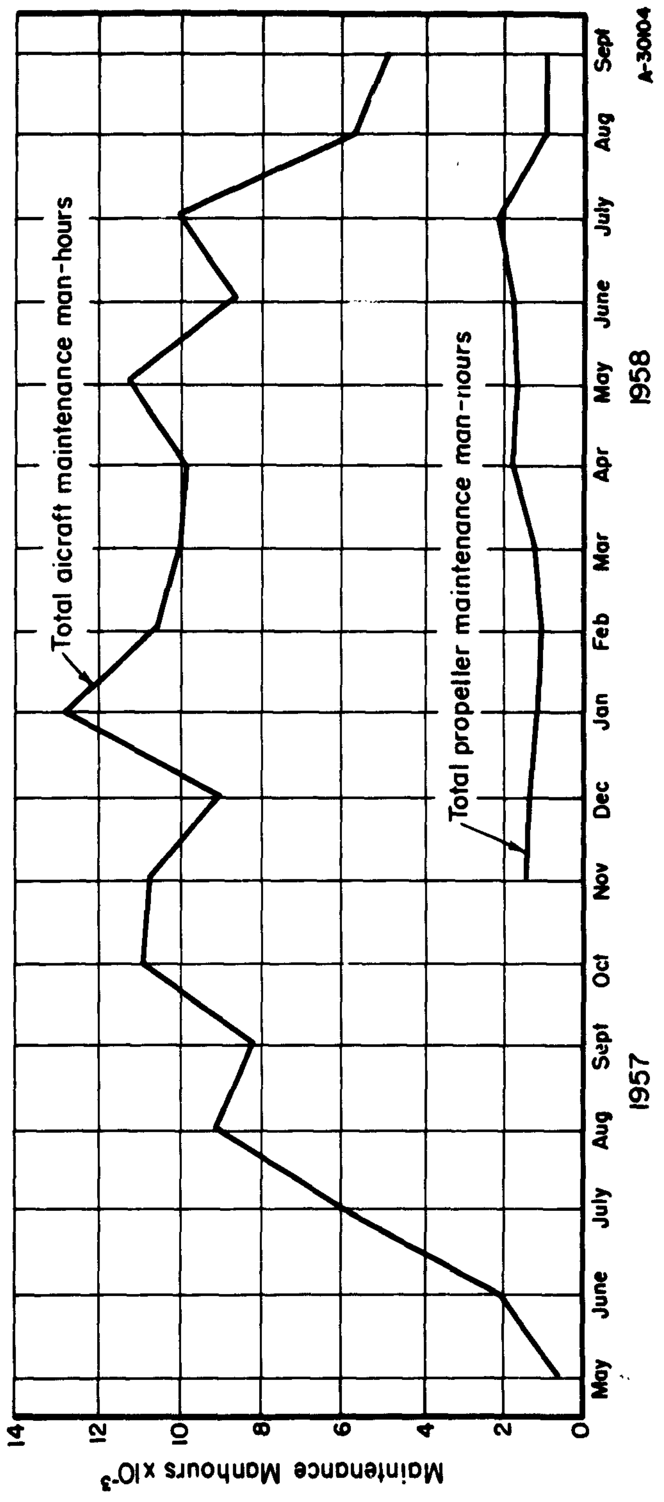

FIGURE 30. PROPELLER AND AIRCRAFT MAINTENANCE MAN-HOURS EXPENDED ON THE TURBOPROP AIRCRAFT

expended on jobs as listed in Table 4. The DD Forms 781-2 contain only a record of the work done as contained in the job standards, plus propeller servicing. Consequently, the total number of propeller man-hours recorded on the DD Forms 781-2, as presented graphically on Figure 31, represents approximately 15 per cent of the total man-hours expended by propeller maintenance personnel.

The propeller maintenance actions reported on the DD Forms 781-2 were tabulated according to the time required to perform the particular action. It was found that approximately 1/3 of the actions required 1 man-hour or less, 6/10 required 2 man-hours or less, and 8/10 required 4 man-hours or less. This indicates that most of the malfunctions experienced with the propeller are minor and require a minimum amount of maintenance action.

Figure 32 shows graphically the history of the maintenance man-hours per flying hour for the period of the data-collection program. Total maintenance man-hours was taken from the DD Forms 781-2 but the propeller maintenance man-hours, which include the miscellaneous man-hours expended on the props, were received from the personnel in the Reports and Analysis Section.

Figure 33 shows the history of the aircraft flying hours per month and also the total number of flights per month. The average flight for the period studied was approximately 2.68 hours. The decline in both aircraft hours and number of flights recorded after June, 1958, is experienced as a result of the assignment of many of the aircraft to temporary duty, with subsequent absence of DD Form 781-2 as a source of data. A more accurate indication of the aircraft flying rate is shown on Figure 34, which is a record of the total flying hours per aircraft flying. It will be noted that the trend in average number of flying hours per month per aircraft is upward until a level of approximately 40 hours per month per aircraft is reached. This corresponds to average experience for transport-type aircraft units maintaining 70 per cent availability and 8 per cent utilization rates.

Personnel in the Report and Analysis Section also provided a record of the rate of Aircraft Out of Commission for Parts. This was calculated monthly by dividing the total number of AOCP hours by the number of assigned aircraft hours. This is shown graphically in Figure 35.

Analysis of Operational Data

The importance of complete and accurate knowledge of operational conditions in the environment in any evaluation of operational reliability is evident from the preceding tables and graphs. During the data-collection period from January to September, 1958, operational demand in terms of

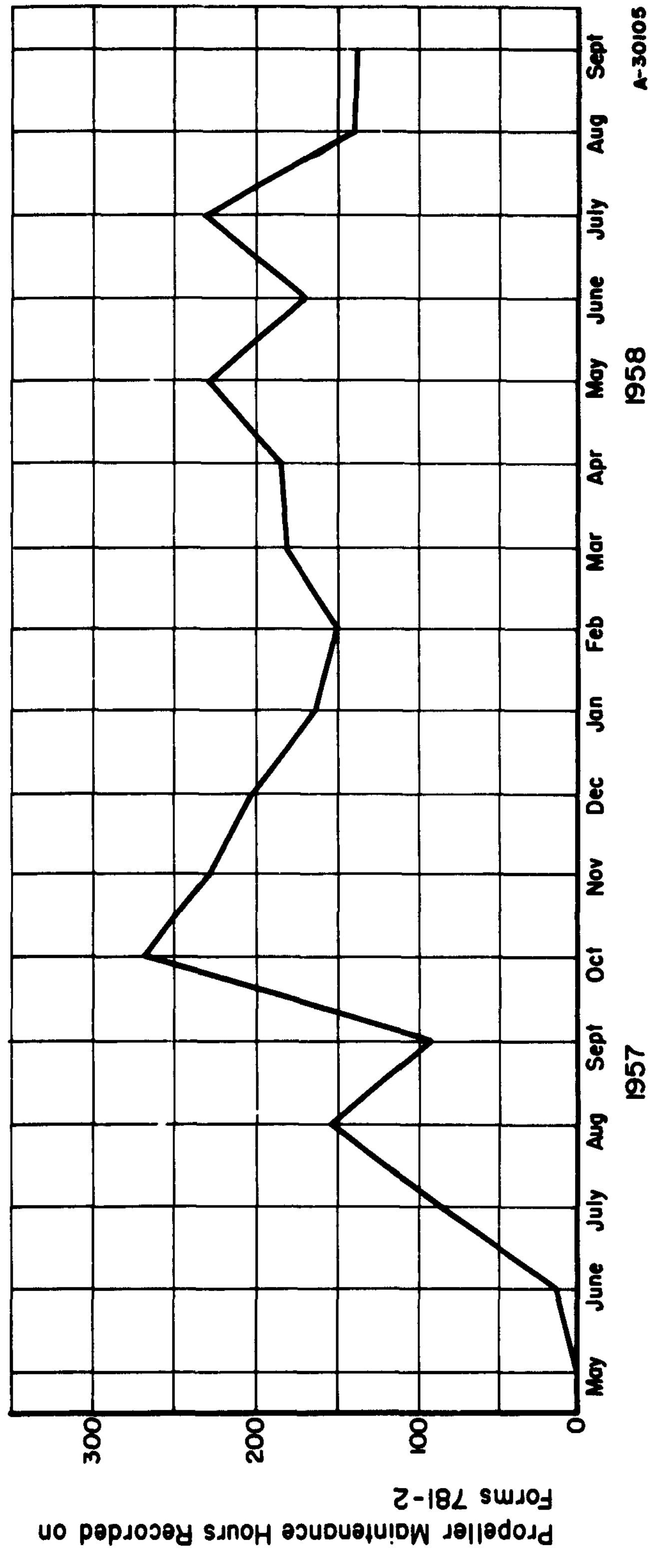

FIGURE 31. PROPELLER MAINTENANCE MAN-HOURS AS RECORDED ON DD FORMS 781-2

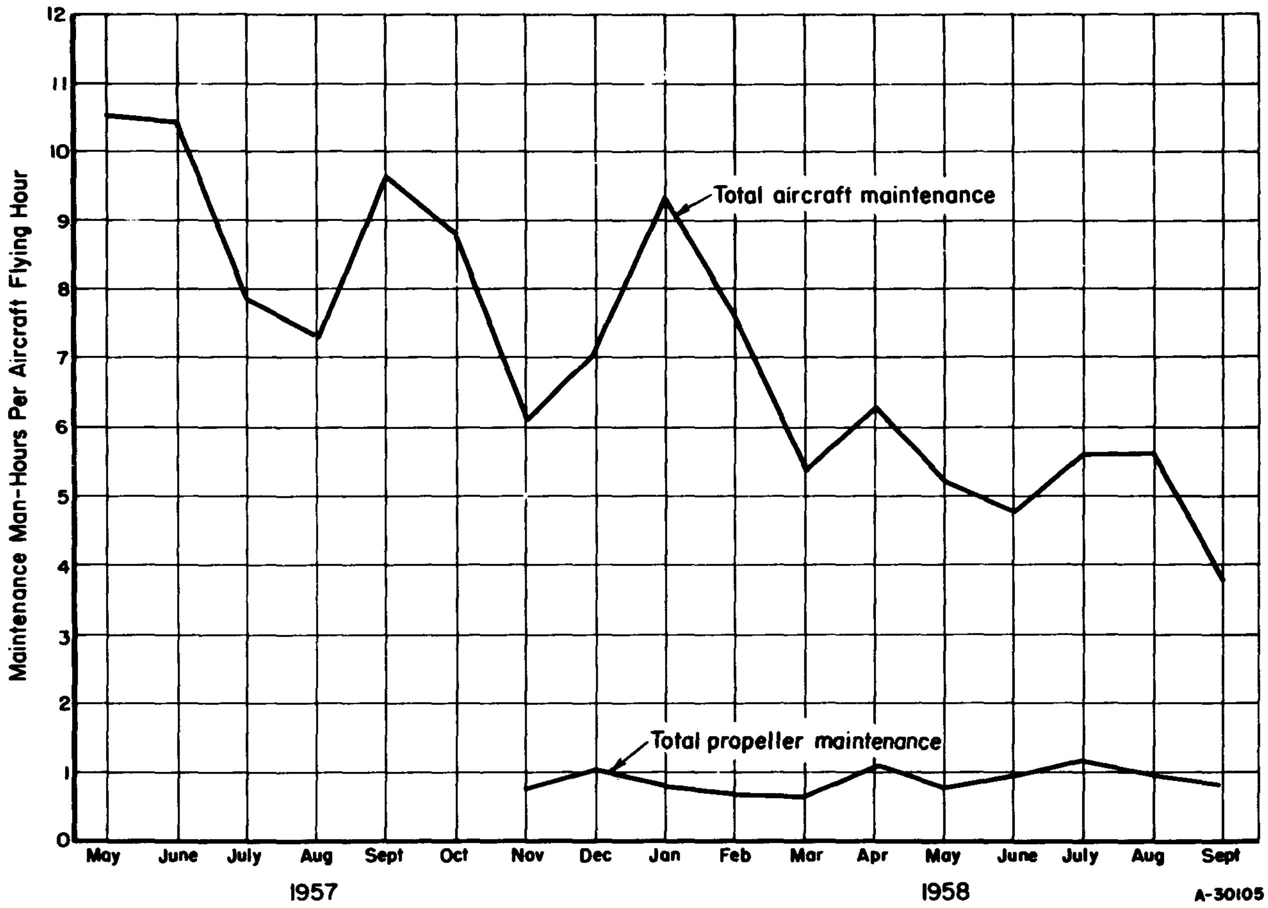

FIGURE 32. AIRCRAFT MAINTENANCE MAN-HOURS AND PROPELLER MAINTENANCE MAN-HOURS PER AIRCRAFT FLYING HOUR

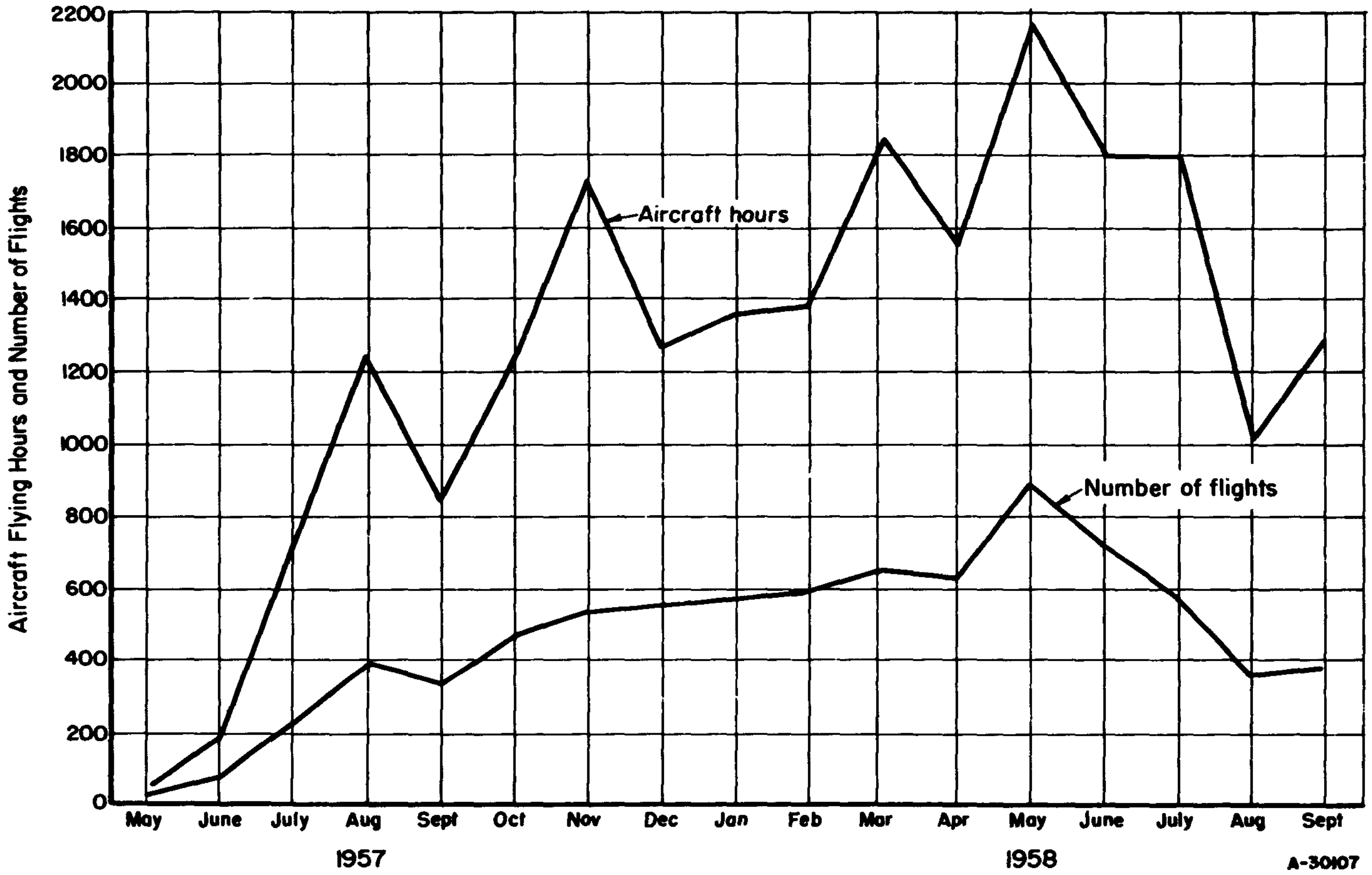

FIGURE 33. AIRCRAFT FLYING HOURS AND NUMBER OF FLIGHTS DURING THE DATA-COLLECTION PROGRAM

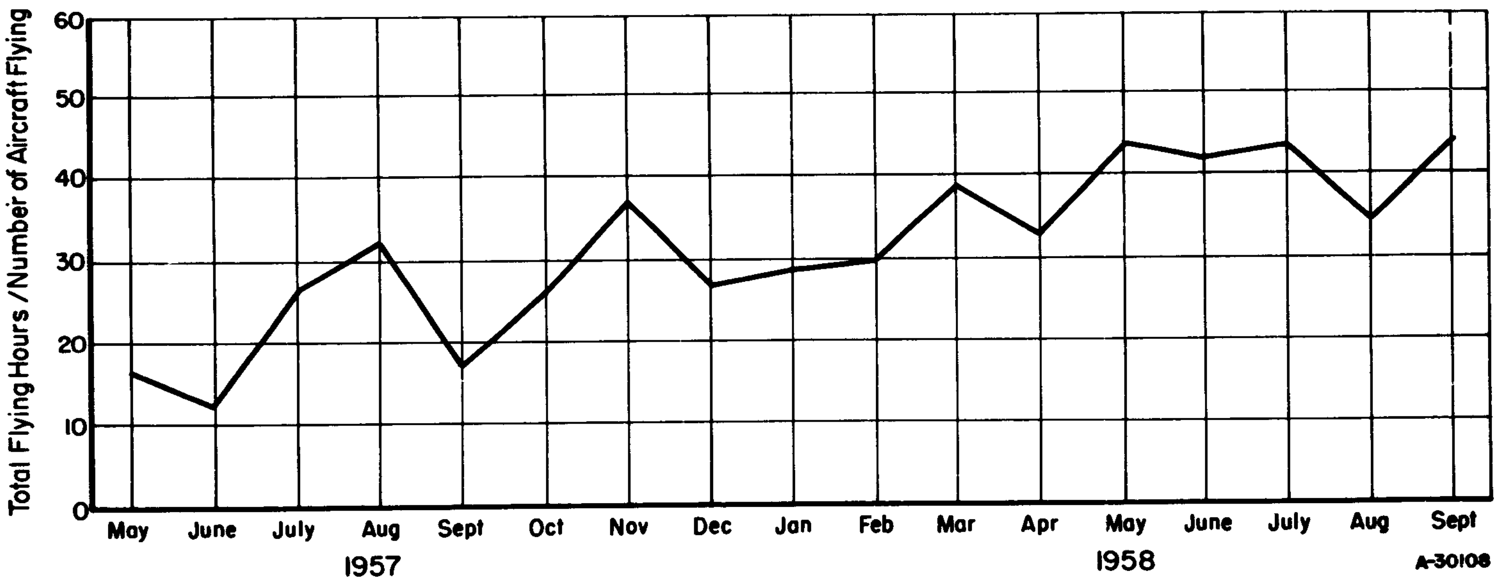

FIGURE 34. AVERAGE NUMBER OF FLYING HOURS PER AIRCRAFT FLYING DURING THE DATA-COLLECTION PROGRAM

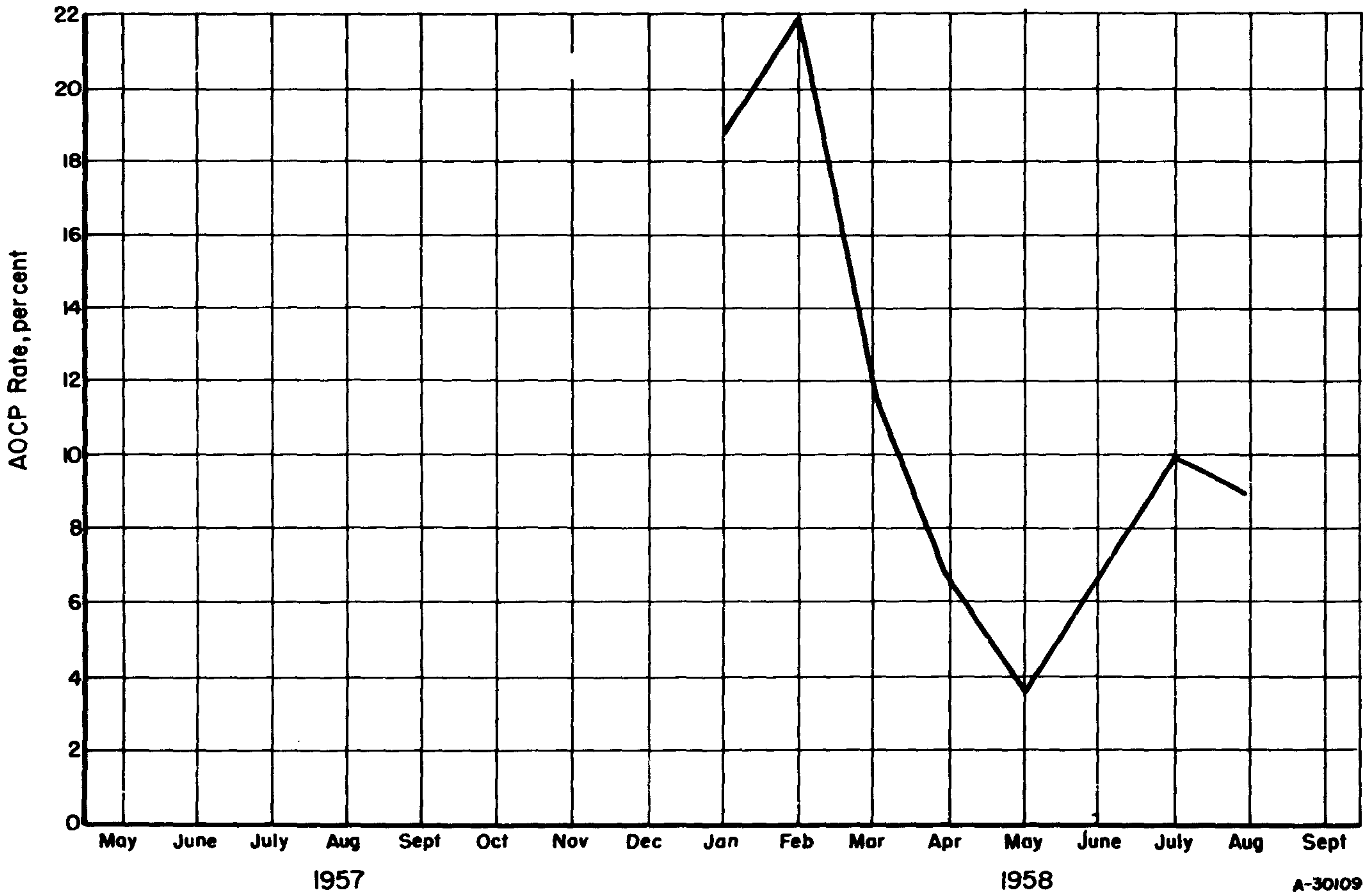

FIGURE 35. AIRCRAFT-OUT-OF-COMMISSION-FOR-PARTS RATE FOR PART OF THE DATA-COLLECTION PROGRAM

aircraft flying hours and number of flights varied irregularly from a moderate low value in January of 1360 hours in 580 flights to a peak in May of nearly 2200 hours in 900 flights. The abnormal dropoff in August was due to temporary off-Base assignment of aircraft and consequent decline in the number of aircraft for which data were available.

During this same period, AOCP rate dropped from 22 per cent to below 10 per cent. Flying hours per aircraft flying increased unsteadily from less than 30 to more than 40 hours per month. The number of propeller malfunctions per 1000 hours of propeller time varied from 5.8 to a high of 9.7 in August. At the same time, the number of premature removals of propeller components reflects the improvement in ATE reliability early in the period, followed later in the period by a deterioration in the regulators. Despite this variation in operational environment, propeller maintenance man-hours expended remained at less than 20 per cent of total aircraft maintenance man-hours for most of the period. From the operational simulation presented in another section of this report, it appears that the number of propeller maintenance man-hours available would have been sufficient to sustain an order-of-magnitude increase in propeller malfunction rate without serious degradation of operational performance.

An accurate picture of the reliability of propellers and their controls in a dynamic operational environment can be obtained by determining the values of reliability parameters, such as hazard rate. In this analysis, the total number of propeller malfunctions sustained is representative of the propeller control performance, inasmuch as the majority of such malfunctions were those having to do with control elements.

In Table 5, the reliability of the propellers and controls is shown in terms of the hazard rate, i.e., the probability of malfunction in the next flying hour for the total population. It must be recalled that the hazard rate is defined as the number of malfunctions divided by the number of exposure hours in a given unit of time. In Table 5, the basic 100-hour time unit is obtained by dividing the propeller population into 100-hour age groups.

The hazard rate for complex equipment is generally expected to remain fairly constant during the period of low-frequency random failures. The propeller control observed during this study is seen to exhibit a nearly constant hazard rate throughout the first 400 hours of operation. However, in the 400-to-500-hour age group the probability of malfunction increases rather suddenly by nearly a factor of 2. This is a strong indication that some element of the propeller is sustaining a wear-out condition. Since the 400-to-500-hour propeller age group does not correspond to any particular calendar time, there is not necessarily any direct relationship between the observed ATE or regulator malfunctions and the noted increase in hazard rate.

TABLE 5. RELIABILITY OF PROPELLERS AND CONTROLS

Propeller Age Groups, hours:	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Total Population								
Number of Propellers Operating This Period	212	211	205	199	126	52	20	1
Propeller Hours Flown	20860	20750	20130	14380	7190	2760	520	30
Number of Malfunctions	144	152	125	98	87	25	5	0
Hazard Rate Per Hour	.00690	.00733	.00621	.00682	.01210	.00906	.00962	--
Original Population Only[a]								
Number of Propellers Operating This Period	210	193	181	155	94	31	14	0
Propeller Hours Flown	19710	19520	16390	10630	4810	1750	270	0
Number of Malfunctions	135	131	103	76	65	16	(1)[b]	0
Hazard Rate Per Hour	.00685	.00671	.00628	.00715	.01351	.00914	(.00370)	0
Number Premature Removals	14	10	16	17	14	3	1[b]	0
Removal Rate Per Hour	.00071	.00051	.00098	.00160	.00291	.00171	.00370[b]	0
Cumulative Premature Removals	14	24	40	57	71	74	75[b]	0
Cumulative Propeller Hours	19710	39230	55620	66250	71060	72810	73080	0
Survival Rate Per Hour	.00071	.00061	.00072	.00086	.00100	.00102	.00103	0
Replacement Population Only[c]								
Number of Propellers Operating This Period	23	41	60	48	41	19	(b)	(b)
Propeller Hours Flown	810	1910	2780	4080	2520	1330	(b)	(b)
Number of Malfunctions	9	21	22	22	22	9	(b)	(b)
Hazard Rate Per Hour	.01111	.01010	.00791	.00539	.00873	.00677	(b)	(b)

(a) Propellers that have not been removed for malfunction since new.
(b) Incomplete records.
(c) Propellers that have been removed for repair and reinstalled.

The total population of propellers contains both original and replacement equipment. It was suspected that these two distinct groups would exhibit different malfunction patterns, and this is indeed evident. In Table 5, the original population exhibits a reasonably stable hazard rate of approximately 0.007 during the first 400 hours. This is equivalent to one failure or malfunction expected for each 143 hours of operation. In the 400-to-500-hour age group, hazard rate increases to nearly 0.014, or an expectancy of one failure for each 72 hours of operation.

Replacement propellers exhibit a distinctly different pattern of malfunction. In the first 200 hours after reinstallation, the replacement propellers sustain a hazard rate of approximately 0.01 equivalent to one expected failure in each 100 hours. Thus, the replacements are prone to fail at least once shortly after installation. Thereafter, the hazard rate declines to the apparent normal average rate for this propeller model.

After plotting the hazard rate of malfunction graphically in Figure 36 for the original, replacement, and over-all population of propellers, the apparent incidence of malfunction in the 400-to-500-hour bracket is at once evident. The vertical or hazard rate scale is exaggerated to emphasize the small changes in hazard rate. At the average flying rate of 40 hours per month or less, an increase of 100 hours in propeller age is the equivalent to an elapsed time of more than 2-1/2 months of operation.

It should be pointed out that the terms "failure" and "malfunction" are not synonymous in the case of propeller controls. The term "failure" implies that some element of the system must be replaced in order to resume normal operation. Well over half of the propeller troubles constitute malfunctions, rather than failures, because the repair and adjustment involved does not require replacement of parts.

The number and rate of premature removals of propellers in the original population indicate those malfunctions or failures that could not be corrected through routine repair and adjustment. In many instances, parts replacement was not necessary. The removal or probability of survival per hour shows an upward trend to the peak of 0.003 approximately in the 400-to-500-hour age group, equivalent to one removal expected in each 340 hours of operation. Removals for inspection are specifically excluded from this calculation. Accumulating the premature removals and relating them to accumulative aging of the propeller leads to an estimate of the survival rate or probability of survival as the propeller and its control continued to age. In Figure 37, the probability of survival, S_t, is plotted against time for the observed propeller system. Assuming that the survival curve takes the form of a Poisson distribution, in accordance with random failure conditions, the cumulative survival rate can be used to estimate the half-life or two-thirds life of the equipment. In practice, the two-thirds life is often equivalent to the mandatory overhaul time. In Figure 37, it can be seen that the half-life of the equipment observed is approximately 700 hours and the two-thirds life is approximately 1100 hours. This means that

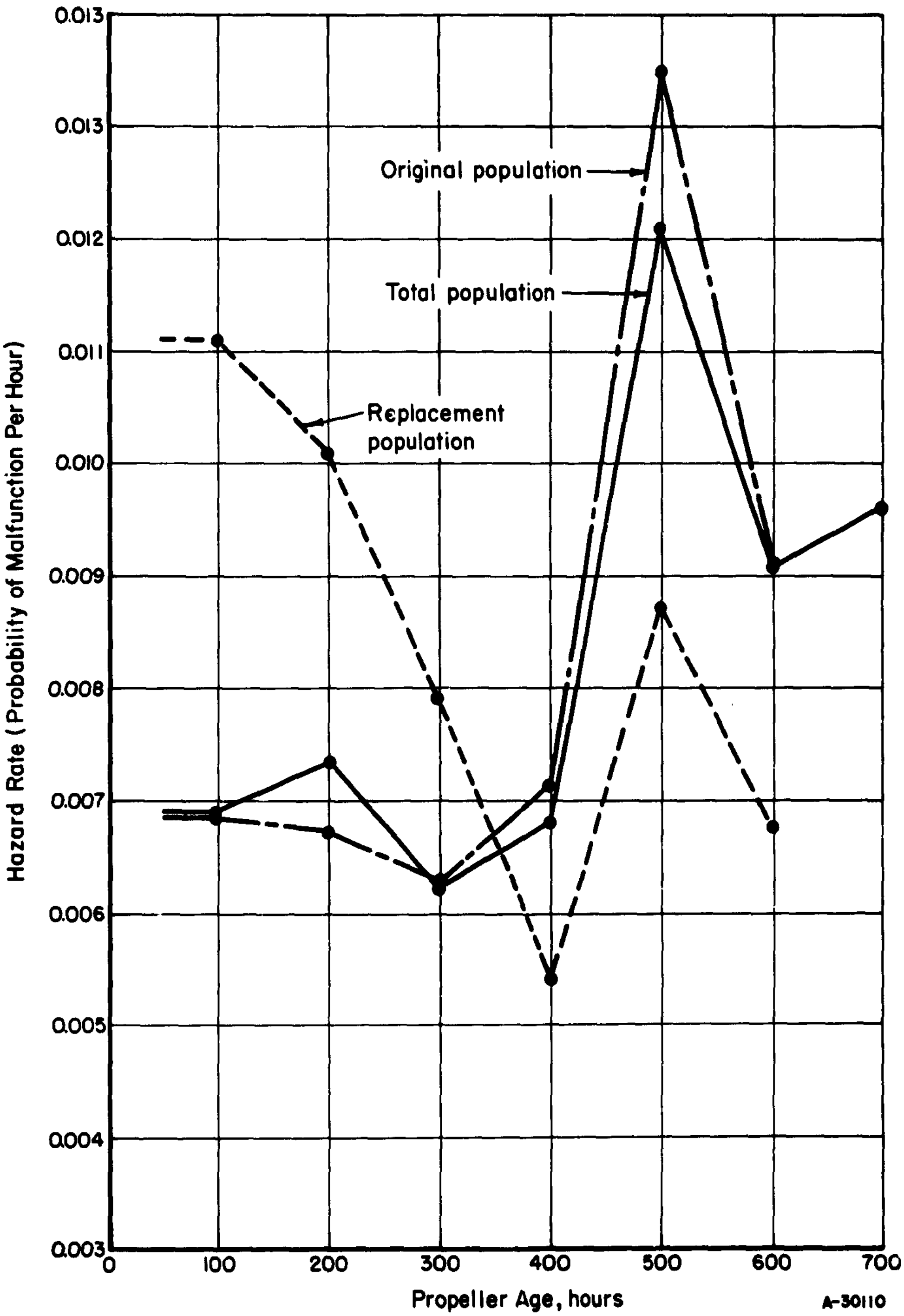

FIGURE 36. HAZARD RATE OF MALFUNCTION EXPERIENCED BY PROPELLERS AND CONTROLS

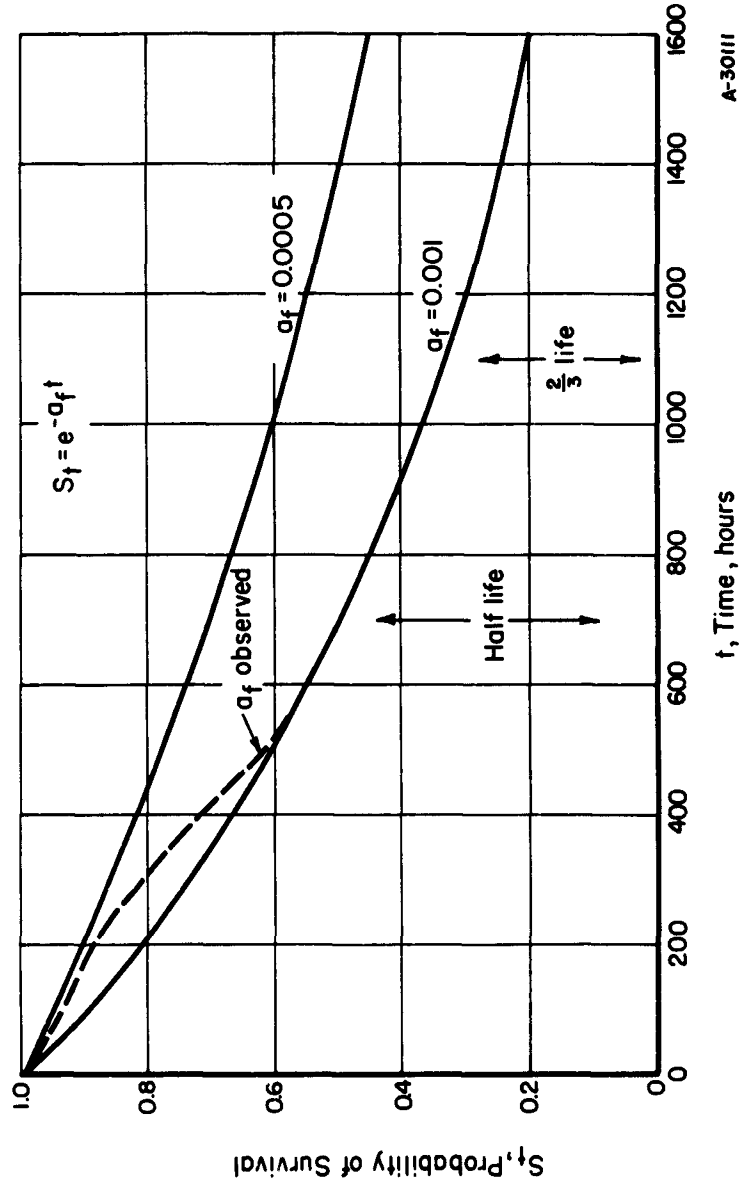

FIGURE 37. PROBABILITY OF SURVIVAL DERIVED FROM FIELD DATA

at the 700-hour point it can be expected that half of the present original population will still be in operation. In the early life of the propeller and its control, a running estimate of probability of survival would have indicated that the half-life of the system would be somewhere between 1300 and 1400 hours. The decline of between 200 and 500 hours indicates a deterioration in the reliability of the system during this particular period of aging. This is not unusual. It represents the more or less typical degradation of reliability of complex equipment experienced under operational conditions, as contrasted with the less severe and less variable flight-test environment.

APPENDIX E

COMPUTER SIMULATION AND ANALYSIS OF OPERATIONAL RELIABILITY

APPENDIX E

COMPUTER SIMULATION AND ANALYSIS OF OPERATIONAL RELIABILITY

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APPENDIX E

COMPUTER SIMULATION AND ANALYSIS OF OPERATIONAL RELIABILITY

The reliability of aircraft and their components under operational conditions is dependent upon a great many factors other than the inherent reliability of the equipment. The technical definition of reliability recognizes this fact when it states that reliability is a function of assigned task and environment. The term "assigned task" refers to frequency and duration of flight as well as to the severity of use relative to the intended purpose of the equipment. The term "environment" takes into account the immediate physical environment in which the equipment is operating and includes a great many other factors contributing to operational success, such as maintenance availability and effectiveness of logistic support.

One approach to evaluating situations containing a substantial number of stochastic variables is to simulate the interaction of these variables with a computer. There are some advantages and some disadvantages to this approach, both well substantiated in the literature on simulation. Although its limitations are recognized, this method is often, as in this case, the only one that permits simultaneous consideration of the many variables involved.

Two problems were simulated during this study, differing in point of interest and in degree of complexity. The first simulation was undertaken to observe the effects of several variables on the status of a propeller-control population as that population aged in operational use. The variables of interest included the aging process, hazard rate, type of repair (field maintenance or overhaul), and fluctuation in number of mandatory overhauls as a function of time.

The second simulation was undertaken to determine the effects of operational flying load, hazard rate, malfunction repair requirements, and maintenance capability on the operational capability of a defined USAF unit. The basis for this simulation was field data obtained on the operations of turboprop aircraft during this study. One objective of this simulation was to show the relative importance of propeller malfunctions as they affect unit capability in comparison with other events occurring in the general situation. Another objective was to determine the relative importance of malfunction rate and maintenance repair time as causative factors in producing propeller maintenance queues.

The starting point for operational analysis is to determine what is meant by "operations". In this study, the term "operations" is represented by the generalized flying cycle shown in Figure 38. This is the operational cycle for flying equipment of flight systems at the unit or Base level. The

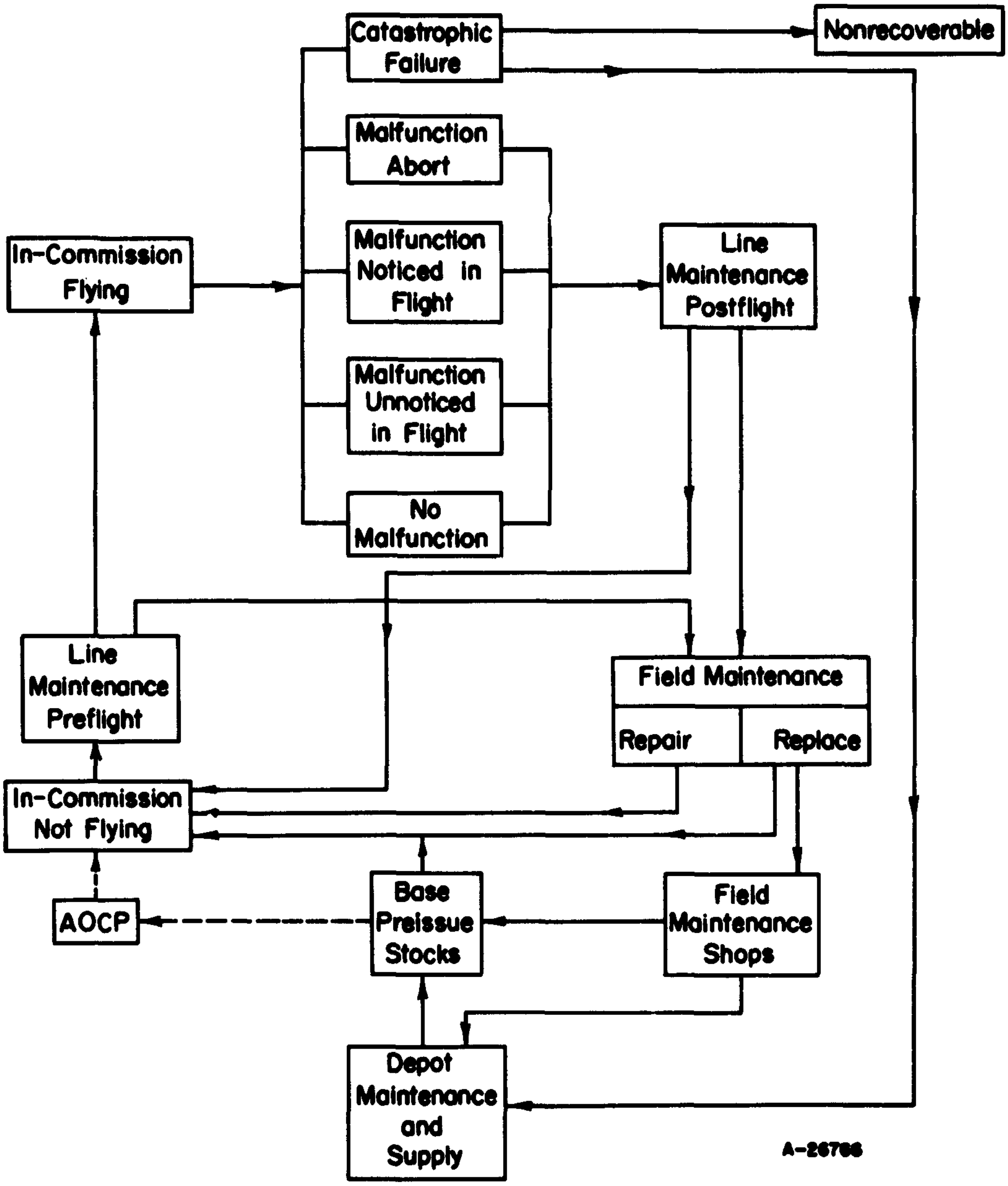

FIGURE 38. A GENERALIZED FLYING CYCLE

paths in Figure 38 indicate the movement of the system through the flying maintenance cycle. The occurrence of a malfunction and the nature of that malfunction determine the path the system follows through the cycle. If the hazard rate of the systems in operation is constant (i.e., an exponential failure distribution), then the expected number of malfunctions per unit time is often calculated by an approximation $m = N(1 - e^{-at})$, where N is the number of systems flying during a given period of time, a is the hazard rate or the probability that a system will fail in an increment of time (i.e., 1 hour), and t is the expected number of flying hours per system during the given period of time. For example, if 100 propeller controls with a hazard rate of 0.01 in the next hour fly an average of 4.0 hours per day, the expected number of malfunctions by this method of calculation would be approximately 4. If the hazard rate is increased to 0.1, the expected number of failures increases to 33.

Although this seems an easy method, it is useful only if the many assumptions upon which it is based are true. For example, it must be assumed that all systems available are, in fact, operated and that all malfunctions that may occur have an equal likelihood of occurrence. Neither of these assumptions can be justified satisfactorily. The method suffers also from the use of expected values in estimating short-term effects. The dispersion and irregularity in the frequency of malfunctions is a major factor in determining operational capability. Further, this method says nothing about the maintenance capability to repair these malfunctions or the influence of assigned mission or observed hazard rate on operations.

The First Simulation

A population of 400 new assemblies is introduced into service over a period of months at a prescribed rate. The introduction rate need not be uniform, but may vary from month to month. There is a fixed utilization policy for flying-hour rate and mandatory overhaul. The nature of these assemblies is such that, if they are inoperable, the aircraft on which they are mounted cannot operate. The following assumptions are made regarding the operation:

(1) The aging process or accumulation of flying hours each month among the assemblies is approximated by a Poisson distribution:

Hours	Per Cent	Hours	Per Cent	Hours	Per Cent
0-25	4.2	75-100	25.8	150-175	2.7
25-50	15.5	100-125	17.2	175-200	0.6
50-75	25.8	125-250	8.1	200-225	0.1

(2) All assemblies have an equal likelihood of flying in any increment of time.

(3) Repair or replacement of an assembly for malfunction or mandatory overhaul is an instantaneous process and AOCP is zero.

(4) In-service inventory remains constant during each monthly period and is equal to the cumulative number of assemblies introduced, excluding spares.

(5) Failure pattern is exponential with time, and hazard rate remains constant.

With these simplifying assumptions, the time history of the population can be constructed by means of a computer program to find the expected number of failures and removals for mandatory overhaul on a monthly basis.

The results of several cases are summarized in Table 6 and are shown graphically in Figure 39. In Case I, the hazard rate ($p_f = 0.0005$) and constant rate of introduction into service permit the cycle to stabilize relatively fast at a ratio of expected failures to expected mandatory overhauls of about 1:5. Cases II and III, where $p_f = 0.001$, result in a ratio of expected failures to expected mandatory overhauls at about 1:2.5 but show no significant effect from the fluctuating introduction rate other than a slight delay in reaching stabilization. It must be remembered, however, that the assumptions are such that the stability produced is somewhat artificial, and that the present model of the situation needs considerable refinement. To observe the effect of an increasing mandatory overhaul time, Case IV permits the original 300-hour limit to double after 6 months of operation. This change in operating conditions further extends the settling time and brings about a 1:1 ratio between expected failures and expected mandatory overhauls. The last set of conditions, Case V, raises the hazard rate by an order of magnitude to $p_f = 0.01$. This condition is sufficiently severe so that the flying-hour rate has to be reduced. The 34:1 ratio of expected failures to expected mandatory overhauls shows that few assemblies survive to mandatory overhaul under this set of conditions.

Referring again to Figure 38, it is obvious that the number of assemblies sent to the overhaul Depot depends upon the fraction of failures repairable by the operating unit and is the sum of the nonrepairable failures and the removals for mandatory overhaul. On-Base repair capabilities depend upon the nature and severity of the failure and the support equipment available. The number of spare assemblies (N) required to replace those that fail (F) or are removed for overhaul (O) is linearly related to the elapsed time in months in the overhaul pipeline (M) and can be written as $N = \frac{M}{30} [O + (1-a) F]$, where O and F are stochastic variables and a is the fraction of failures repaired on the Base.

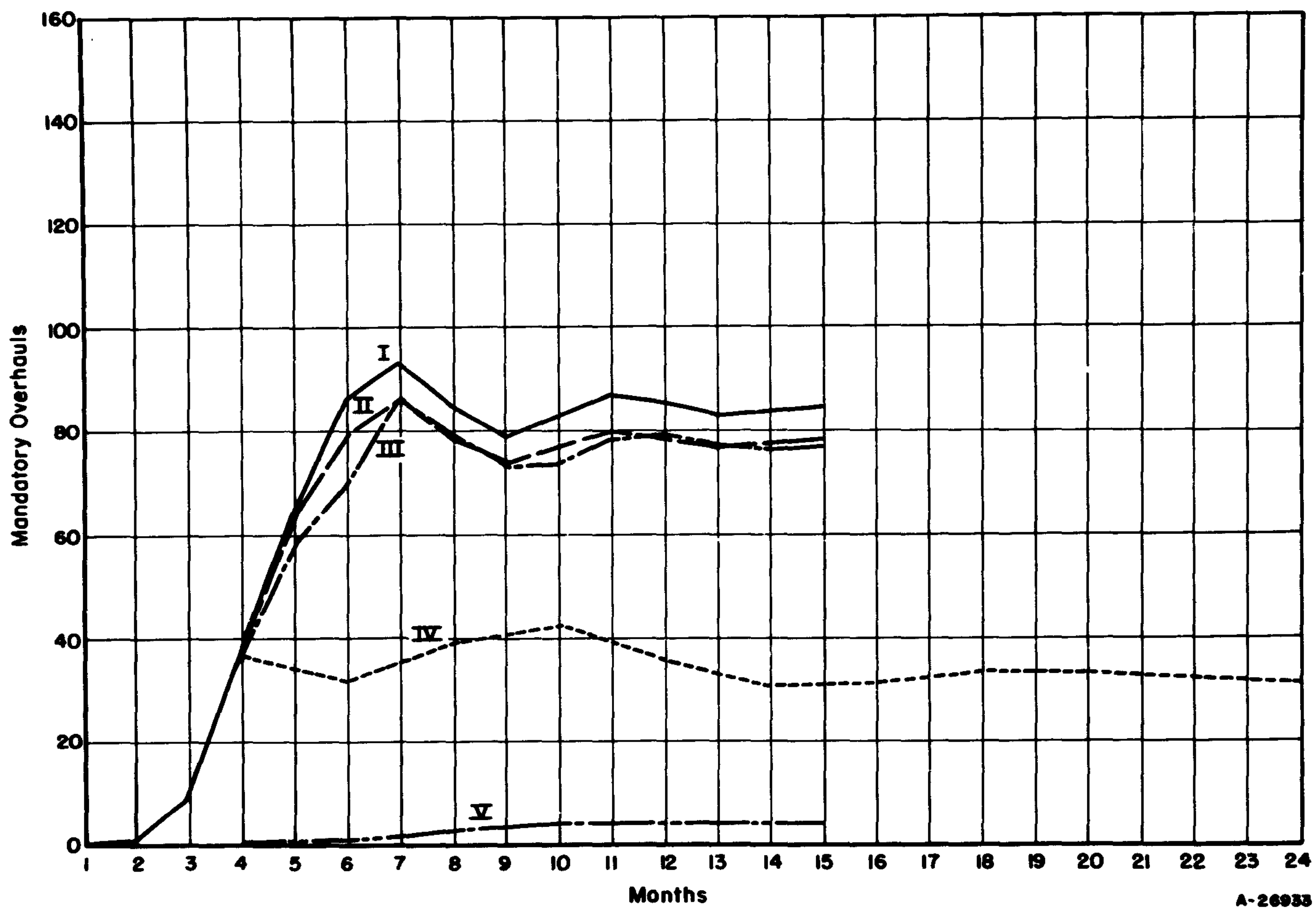

FIGURE 39. MANDATORY OVERHAULS PER MONTH FOR 400 ASSEMBLIES UNDER SEVERAL ASSUMED OPERATING CONDITIONS

TABLE 6. SUMMARY OF MONTHLY FAILURES AND MANDATORY OVERHAULS

	Case I			Case II		
Mandatory Overhaul, hours:	300			300		
Rate of Introduction Into Service Per Month:	100			100		
Hazard Rate, p_f:	0.0005			0.001		
Average Flying-Hour Rate, hours per month:	81.25			81.25		
Month	Failures	Mandatory Overhauls	Total	Failures	Mandatory Overhauls	Total
1	4.0	0.0	4.0	8.0	0.0	8.0
2	8.1	0.3	8.4	16.1	0.3	16.4
3	12.1	9.5	21.6	24.1	9.2	33.3
4	16.2	39.5	55.7	32.1	37.1	69.2
5	16.2	69.3	85.5	32.1	63.5	95.6
6	16.2	86.7	102.9	32.1	79.1	111.2
7	16.2	93.4	109.6	32.1	85.8	117.9
8	16.2	85.4	101.6	32.1	78.5	110.6
9	16.2	79.6	95.8	32.1	73.7	105.8
10	16.2	83.2	99.4	32.1	76.9	109.0
11	16.2	86.8	103.0	32.1	79.7	111.8
12	16.2	85.6	101.8	32.1	78.5	110.6
13	16.2	83.4	99.6	32.1	76.9	109.0
14	16.2	83.7	99.9	32.1	77.3	109.4
15	16.2	84.9	101.1	32.1	78.1	110.2
16						
17						
13						
19						
20						
21						
22						
23						
24						
25						
26						

FOR 400 ASSEMBLIES UNDER SEVERAL ASSUMED OPERATING CONDITIONS

Case III			Case IV			Case V		
300			300 for 6 months, then 600			300		
50-150-50-150 over 4 months			100			100		
0.001			0.001			0.01		
81.25			81.25			40.0		
Failures	Mandatory Overhauls	Total	Failures	Mandatory Overhauls	Total	Failures	Mandatory Overhauls	Total
4.0	0.0	4.0	8.0	0.0	8.0	36.1	0.0	36.1
16.1	0.3	16.4	16.1	0.3	16.4	72.2	0.0	72.2
20.1	9.4	29.5	24.1	9.2	33.3	108.3	0.0	108.3
32.1	37.4	69.5	32.2	37.1	69.3	144.3	0.1	144.4
32.1	59.2	91.3	--	--	--	144.3	0.3	144.6
32.1	69.4	101.5	32.2	31.8	64.0	144.3	0.9	145.2
32.1	86.5	118.6	--	--	--	144.3	1.8	146.1
32.1	83.9	116.0	32.2	39.6	71.8	144.3	2.9	147.2
32.1	73.9	106.0	--	--	--	144.3	3.8	148.1
32.1	74.3	106.4	31.8	42.9	74.7	144.3	4.3	148.6
32.1	79.2	111.3	--	--	--	144.3	4.5	148.8
32.1	79.7	111.8	31.8	35.9	67.7	144.3	4.5	148.8
32.1	77.3	109.4	--	--	--	144.3	4.4	148.7
32.1	76.7	108.8	31.8	31.0	62.8	144.3	4.3	148.6
32.1	77.9	110.0	--	--	--	144.3	4.3	148.6
32.1	78.3	110.4	31.8	31.4	63.2			
32.1	77.8	110.9	--	--	--			
32.1	77.5	109.6	31.8	33.6	65.4			
			--	--	--			
			31.8	33.6	65.4			
			--	--	--			
			31.8	32.6	64.4			
			--	--	--			
			31.8	32.6	64.4			
			--	--	--			
			31.8	33.0	64.8			

With the linear relation plotted in Figure 40 for a 45-day overhaul pipeline and values for O and F previously obtained, the influence of on-Base repair capability is evident. In Cases I and IV, where relatively fewer failures occurred in comparison with removals for mandatory overhaul, the influence of on-Base capability is minor. For Case V, however, where a very high proportion of the total maintenance actions were irregular removals for failure, on-Base repair capability has a marked effect. Several interpretations are possible here. First, field maintenance may be assumed to have the capability for complete disassembly of the equipment and repair of all types of failure. Second, field maintenance has only a limited repair capability, but most or all of the failures are of a minor nature and within the repair capability. If the repair capability is defined, then the expected total number of assemblies required becomes dependent upon the nature of the failure and thus can be related to the reliability of the assembly. In this analysis, availability was held constant at 100 per cent by assuming that repair and replacement were accomplished instantaneously. If the total number of assemblies was held constant and lag times for repair and replacement were permitted, AOCP rates and availability would become dependent variables.

The Second Simulation

From the preceding work, it is evident that the reliability of propeller controls in an operational environment is not readily determinable without a more complete simulation of the other important variables in the environment. In the second simulation, a determined effort was made to inject more realism into the simulation without undermining the validity of the results. The flying load imposed upon the operational unit, frequency and duration of flights, maintenance repair times, and certain other operational parameters were permitted to vary. In the following paragraphs, the procedures and results of the second simulation are described.

General Description of the Simulation

The system to be simulated is the flight operations and maintenance of an operational unit equipped with four-engine, turboprop aircraft and operating in the ZI. Only that portion of unit operations having to do with flying and maintaining the aircraft is considered. The objective is to determine the influence of propeller-control reliability on the operational capability of the unit.

The simulation is dynamic but is carried on in nonreal time. The unit receives mission assignments and allocates aircraft to those missions within the limits of its capability. The missions are flown at various times of the day and may be aborted or may sustain a failure. In accordance with probability of occurrence of such events, propeller-control failures are

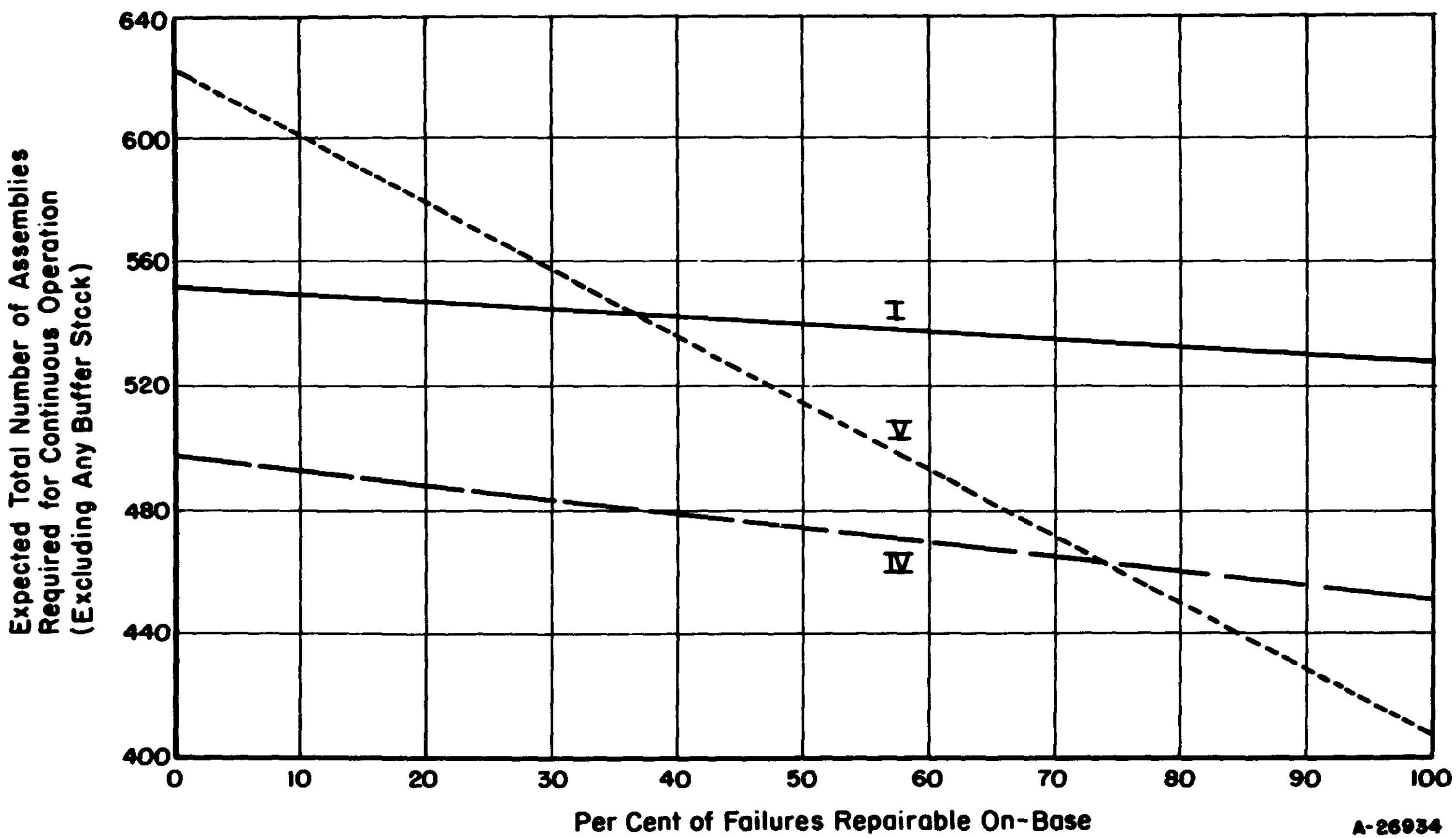

FIGURE 40. INFLUENCE OF ON-BASE REPAIR CAPABILITY IN TOTAL STOCK REQUIREMENTS, ASSUMING A 45-DAY OVERHAUL PIPELINE

identified and treated separately. Both general maintenance and propeller maintenance may be accomplished following the flights or aborted missions. Maintenance service time and the availability of spare parts determine the availability of aircraft for subsequent missions. The simulation follows the flying and maintenance operations for a period of 30 days, recording the daily history and the status at 0800 each day.

Dynamic simulation of unit operations in this fashion provides a basis for determining the relative influence of propeller failure on operational capability.

Assumptions

The simulation is based on certain assumptions that fix the quantitative rclations in the analysis. These assumptions are as follows:

(1) The number of aircraft in the systems is fixed at 50 (catastrophic failure is neglected).

(2) Flight operations and maintenance were conducted on an hourly basis and flight and repair schedules are prescribed in 1-hour increments.

(3) All flights originate and terminate at one Base. All maintenance activities are conducted at this Base. (Multiple-Base operations could be considered with only minor modifications in the maintenance scheduling.)

(4) Missions are flown daily on a 5-day-week basis. The number of missions to be flown during any 24-hour period is generated at 0800 each day. No missions are generated for the 2-day weekend, but missions assigned on the fifth day that have not been flown by 0800 on the sixth day are reassigned, thus simulating a reduced operational requirement during this period.

(5) A mission abort is assumed to occur prior to take-off and the mission is not reassigned until the following hour.

(6) The probability of propeller failure per hour is assumed constant and independent of propeller age. The contingency of more than one propeller failure per flight is not considered.

(7) After take-off, the mission is completed, regardless of malfunction of the propeller or of any other system.

(8) General maintenance is required following each mission or abort. Propeller maintenance is required only if the in-flight failure or abort was due to propeller malfunction.

(9) General maintenance and propeller maintenance are conducted on the basis of two 8-hour shifts in a 5-day week. New maintenance work is not assigned on the 2-day weekend, but uncompleted maintenance on the fifth day requires weekend maintenance activity. There is no exchange of personnel between general and propeller maintenance.

(10) The number of men assigned to specific maintenance tasks is fixed and limited to not more than four.

(11) Periodic inspections are performed when aircraft flight time reaches specified values, e.g., 150 and 300 hours. Maintenance required for a 150-hour inspection is assumed to be 24 consecutive hours. Maintenance required for a 300-hour inspection is assumed to be 120 consecutive hours. Inspection is accomplished following the mission that overflies the inspection time.

(12) AOCP is assumed to occur in a fixed fraction of the maintenance actions required. Not more than one AOCP occurs for each maintenance repair job. Maintenance is delayed until AOCP is rectified. General and propeller maintenance channels are regarded as separate, and AOCP is determined for each, independently of the other. Both types of maintenance may be performed on a specific aircraft simultaneously, and progress through the two channels is not necessarily parallel. For example, the aircraft may be in the propeller maintenance queue while general maintenance is in progress.

(13) Maintenance queues are considered to form when the men available are not sufficient to undertake a specific job assignment.

Initial Conditions

At the beginning of the simulation, the aircraft and propellers were aged in accordance with the first simulation program and field experience. It was assumed that half of the aircraft were available at the start of operations at 0800 on the first day, and the other half were distributed in maintenance channels and in flight status.

Parameter Variations

The parameters of interest in the simulation are those that influence operational capability when a propeller failure occurs. The rate at which flying activity produces propeller failures determines the maintenance load. The maintenance counteraction, the rate at which maintenance returns complete aircraft to service, acts in opposition to the rate at which missions can be accomplished and thus generate failures. Hence, the parameters of interest include the flying load, characteristics of equipment, failure in terms of frequency and extent of repair required, and the characteristics of the maintenance force.

Many of the events affecting disposition of aircraft occur at random, for example, propeller failure during flight, or maintenance repair time required for that failure. To express the random nature of these variables, Monte Carlo techniques are used to select specific values from prescribed distributions. Operational data available provide a real although limited basis for determining such distributions. However, these data provide a reasonable indication of the relative frequency of such events and, if the analysis is continued for a sufficient period of time, the repeated application of random selection (Monte Carlo techniques) should produce the same relative frequency of events. Parameter variations introduced in the simulations are summarized in Table 7. The numerical values shown are average or mean values descriptive of the parametric distributions actually employed. Discrete values in the frequency distribution for each parameter are given in appropriate tables later on. Methods of generating the distributions and their use in a computer simulation are discussed later in the context of the computer model.

Results of the Second Simulation

The computer model used to simulate operational use of a turboprop system provides a means for analyzing the effects of propeller reliability on the operational capability of an aircraft system. To analyze the effects on the system performance properly, several measures of effectiveness of propeller reliability were selected. The basic measures of systems performance are as follows:

(1) The number of aircraft in the propeller maintenance queue as a function of time

(2) Aircraft availability

(3) Number of missions accomplished

(4) The number of unassigned propeller maintenance personnel.

TABLE 7. PARAMETER VARIATIONS IN THE SIMULATION

Simulation Run	Average Flying Load, missions per month	Mission Duration, hours	Abort Rate	Probability of Propeller Malfunction Per Flight Hour	Average AOCP Frequency and Duration	Propeller Repair Time and Frequency	Number of Propeller Maintenance Personnel Available Per Shift
1	740	1-6	0.05	0.008	0.1	I	30
2	740	1-6	0.05	0.008	0.3	I	30
3	1480	1-6	0.05	0.008	0.1	I	30
4	740	6	0.05	0.1	0.1	I	30
5	740	1-6	0.05	0.008	0.1	I	30
6	740	1-6	0.05	0.1	0.1	II	30
7	1480	1-6	0.05	0.1	0.1	I	30
8	1480	1-6	0.05	0.1	0.1	II	30
9	1480	1-6	0.05	0.1	0.1	III	30
10	1480	6	0.05	0.1	0.1	III	15
11	740	6	0.05	0.008	0.1	III	15
12	740	6	0.05	0.1	0.1	III	15

The operational effects of propeller reliability on the system, as measured by the above criteria, were analyzed as a function of the following inputs:

(1) Flying load

(2) Mission duration

(3) Probability of propeller malfunction

(4) Probability of propeller AOCP

(5) Propeller repair time

(6) Number of propeller maintenance personnel.

The operational environment in which the propeller is presumed to operate was constructed in terms of the flying schedule, the work-week, and general maintenance variables. The latter may affect aircraft availability and missions accomplished to a greater extent than propeller reliability. This study is not concerned with examining general maintenance, and it is included solely as a means for establishing the context of the operational environment.

The Basic Case

To evaluate the influence of certain reliability parameters on operational capability, it is first necessary to postulate the basic set of parameters and realistic values for a basic case. The parameter values may then be varied and the effect on operational capability observed and analyzed. Numerical values for the parameter variations are listed in Table 7 for Simulation Runs 1 through 12. In the discussion that follows, the simulation runs are analyzed in the order in which they appear in Table 7.

The parameter values shown in Table 7 for Simulation Run 1 constitute the basic case. These values represent a reasonable approximation of the operational situation for a fairly reliable propeller.

The results of the computer simulation for the basic case are shown in Table 8. For the given propeller reliability and maintenance support, mission requirements are completed with very little trouble. Propeller failures of any kind are infrequent, and those that do occur require little maintenance. The number of propeller maintenance personnel available is far greater than the number actually needed to maintain operational capability.

The basic case is shown to illustrate an operational situation in which the propeller is quite reliable and can be returned to service very quickly in the event of failure. It will be noted in this and subsequent cases that the flying load and number of propeller failures per day show a fairly wide fluctuation. These are characteristic of real operations and often make reliability difficult to measure, as well as contributing to the illusion of a constantly changing reliability. It should also be noted that, in this and subsequent cases, Day 1 represents initial conditions, and mission requirements are not actually generated until Day 2.

Influence of High Propeller AOCP Rates

To observe the effect of AOCP rate on operational capability, the probability of AOCP following a flight or an abort was increased from 0.10 to 0.30. This was applied to both general and propeller maintenance channels and represents a general inadequacy of logistic support.

An evaluation of the unit operational capability for this condition is given in Table 9. The increased AOCP rate for propellers does not significantly influence the ability to complete mission requirements. The number of propeller malfunctions remains relatively small, and AOCP does not occur except in the case of a malfunction requiring replacement of a component or part. The over-all capability of the unit is noticeably affected by the AOCP rate in general maintenance. This is not of special interest here except as an environmental background for propeller operation.

The appearance of one aircraft in the propeller maintenance queue on the seventh day (i.e., Sunday) results from the context of the simulation. The model is designed so that maintenance requirements generated on a weekend day are not assigned to the work force until the following Monday. In this and subsequent cases, maintenance generated by aircraft returning from flights on the weekend is held in the appropriate maintenance queue until Monday morning.

Influence of Increased Flying Load

In an attempt to stress the system, the average number of missions assigned per month was increased from 740 to 1480. This would be similar to a unit operating a large number of short flights per day, as in training operations or short-haul cargo operations. The results for this simulation are shown in Table 10.

Under these conditions, a substantial number of the assigned missions cannot be flown. However, the reduced operational capability cannot be attributed to propeller reliability. The requirement for postflight general maintenance is sufficient to limit aircraft availability under these conditions. The number of propeller malfunctions increases because of the increase in number of exposures, but nowhere do aircraft appear in the propeller

TABLE 8. EVALUATION

The Basic

	Operations					Propeller		
Day	Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	30	76	0	0	2	0	0
2	32	32	63	0	0	2	0	2
3	32	32	73	0	1	2	0	0
4	40	40	100	1	1	2	0	0
5	28	28	81	0	1	3	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	30	30	68	0	5	2	0	0
9	22	22	61	0	1	1	0	0
10	32	32	86	0	2	2	0	0
11	48	48	106	0	1	4	0	0
12	34	34	65	0	1	1	0	0
13	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0
15	38	38	97	0	2	4	0	0
16	24	24	51	0	2	0	0	0
17	34	34	78	0	1	3	0	0
18	38	38	103	1	2	0	0	0
19	30	30	77	0	1	3	0	0
20	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0
22	30	30	61	0	2	0	0	0
23	38	38	91	1	1	1	0	0
24	42	42	95	0	1	6	0	0
25	38	38	82	1	0	2	0	0
26	38	38	86	0	1	2	0	0
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	38	38	96	0	2	2	0	0
30	36	36	97	0	2	3	0	0

OF SIMULATION RUN 1

Case.

Maintenance		General Maintenance			
Man-Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	A/C in General Maintenance Queue	AOCP General	Man-Hours of General Maintenance Assigned	Number of General Maintenance Personnel Unassigned
31	30	0	0	587	0
11	30	0	10	773	74
17	30	0	9	672	63
21	30	0	14	882	71
13	30	0	17	595	58
0	30	0	13	0	68
0	30	6	7	0	100
6	30	11	2	820	100
8	30	0	3	350	81
3	30	0	3	497	91
9	30	0	8	898	86
2	30	0	10	551	66
0	30	0	11	0	89
0	30	2	9	0	100
15	30	4	7	1099	100
0	30	0	4	513	68
7	30	0	4	463	80
2	30	0	8	598	87
6	30	0	9	585	76
0	30	0	13	0	83
0	30	6	7	0	100
0	30	8	5	729	100
9	30	0	2	636	80
30	30	0	4	676	81
7	30	0	7	753	83
5	30	0	6	791	70
0	30	0	9	0	64
0	30	3	6	0	100
5	30	8	1	826	100
7	30	0	3	644	75

TABLE 9. EVALUATION

Propeller
All other maintenance parameters

	Operations					Propeller		
Day	Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	32	73	1	1	2	0	0
2	32	27	71	0	2	2	0	2
3	36	17	59	0	2	0	0	0
4	36	25	30	0	1	1	0	0
5	38	13	63	0	1	2	0	0
6	0	0	38	0	0	0	0	0
7	0	36	0	0	0	0	1	0
8	36	20	96	1	1	4	1	0
9	20	40	37	0	2	1	0	0
10	40	29	93	1	2	2	0	0
11	30	24	77	0	1	2	0	0
12	32	8	49	1	1	2	0	0
13	0	0	14	0	0	0	0	0
14	0	44	0	0	0	0	0	0
15	44	30	112	0	0	1	0	0
16	30	36	53	0	2	1	0	0
17	36	26	77	0	2	2	0	0
18	26	32	57	0	1	0	0	0
19	32	0	63	0	0	2	0	1
20	0	0	0	0	0	0	0	1
21	0	40	0	0	0	0	0	2
22	40	28	92	0	3	2	0	1
23	28	34	67	1	1	3	0	1
24	34	30	76	0	3	2	0	1
25	30	27	73	0	2	3	0	0
26	40	13	69	0	0	0	0	0
27	0	0	24	0	1	0	0	0
28	0	30	0	0	0	0	0	0
29	30	40	73	0	0	4	0	0
30	40	38	76	1	3	2	0	0

OF SIMULATION RUN 2

AOCP – 0.30
are the same as for the basic case.

Maintenance		General Maintenance			
Man-Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	A/C in General Maintenance Queue	AOCP General	Man-Hours of General Maintenance Assigned	Number of General Maintenance Personnel Unassigned
30	30	0	0	580	0
13	30	0	17	394	76
0	30	0	24	315	79
3	30	0	30	459	60
7	30	0	29	546	57
0	30	0	25	0	49
0	30	1	19	0	94
23	30	1	13	930	100
2	30	0	21	264	52
8	30	0	20	706	85
8	29	0	26	505	54
5	30	0	28	465	55
0	30	0	29	0	58
0	30	0	23	0	96
9	30	0	11	1301	100
4	28	0	12	537	32
9	30	0	17	498	76
0	30	0	26	429	79
3	30	0	23	574	72
0	30	0	25	0	71
0	30	0	17	0	97
15	30	0	8	815	100
16	30	0	16	707	70
10	30	0	14	550	74
10	30	0	22	455	78
0	29	0	32	606	78
0	30	0	26	0	48
0	30	0	21	0	93
24	30	0	8	972	100
5	28	0	13	540	66

TABLE 10. EVALUATION OF

Average Number of
All other maintenance parameters

	Operations					Propeller		
Day	Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	24	60	0	2	1	0	0
2	61	61	149	1	2	7	0	3
3	67	58	144	0	1	6	0	1
4	73	46	100	0	1	3	0	0
5	67	38	77	0	4	4	0	0
6	0	27	66	0	2	1	0	0
7	0	2	8	0	0	0	0	0
8	59	53	140	0	2	9	0	0
9	67	55	121	0	1	3	0	0
10	53	49	101	1	4	2	0	0
11	63	46	100	0	3	2	0	0
12	71	48	90	0	1	2	0	0
13	0	23	51	0	0	2	0	0
14	0	0	0	0	0	0	0	0
15	56	49	120	0	1	3	0	0
16	67	39	93	0	4	4	0	0
17	79	55	134	0	1	5	0	0
18	73	52	122	0	0	6	0	0
19	63	63	137	0	2	5	0	0
20	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0
22	71	71	176	0	1	5	0	0
23	81	51	119	0	2	4	0	0
24	79	52	136	0	3	5	0	0
25	56	36	85	2	4	3	0	0
26	50	50	119	0	2	2	0	0
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	67	67	142	1	3	5	0	0
30	65	62	148	1	9	5	0	0

SIMULATION RUN 3

Missions per Month - 1480
are the same as for the basic case.

Maintenance		General Maintenance			
Man-Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	A/C in General Maintenance Queue	AOCP General	Man-Hours of General Maintenance Assigned	Number of General Maintenance Personnel Unassigned
24	30	0	0	756	0
16	30	0	10	942	76
21	30	0	10	951	38
26	22	0	10	946	14
22	27	0	13	681	24
0	25	0	15	0	36
0	30	28	16	0	93
31	30	36	10	1780	100
6	26	4	10	922	0
17	28	1	12	861	16
7	30	0	13	1015	13
9	26	0	12	675	11
0	30	0	20	0	31
0	30	22	16	0	100
20	30	31	12	1663	100
27	25	0	12	669	80
23	26	0	12	948	25
24	22	0	10	1063	13
12	24	2	9	1049	13
0	28	1	12	0	9
0	30	4	8	0	89
9	30	6	6	1493	100
4	30	0	8	891	16
32	29	0	10	966	7
18	21	0	16	930	24
6	26	0	13	1119	16
0	30	0	11	0	3
0	30	5	6	0	92
26	30	5	6	1081	100
26	26	0	4	1171	59

maintenance queue at the beginning of daily operations. The number of propeller personnel available are more than adequate to maintain the propellers with the very short service time required. Less than 15 per cent of the available propeller maintenance man-hours are utilized on a one-shift basis.

Influence of Increased Mission Duration

On the fourth simulation run, mission duration was increased to 6 hours and assumed to be the same for all flights. This can be viewed as equivalent to long-haul cargo operations on an overseas run if it is assumed that all maintenance must be accomplished by personnel from the parent Base. The results are summarized in Table 11.

In comparison with the basic case, the effect of increasing the mission duration is essentially an increase in the number of flying hours. There appear to be no adverse effects. Mission requirements are completed on the day assigned with one exception, on Day 27, when one mission was flown late.

Influence of Increased Propeller Malfunction Rates

Up to this point, propeller malfunction rate has been held at the relatively low probability of 0.008 per hour. In this case, this value is increased by an order of magnitude of 0.1 per hour, with the values of all other parameters remaining the same as those in the basic case. An evaluation of this situation is shown in Table 12.

A substantial increase in the number of propeller malfunctions occurs as a result of the increased probability of malfunction. The number of maintenance man-hours required to repair the propeller likewise increases. However, there is no apparent effect on the operational capability of the unit.

It can be concluded from this and previous runs that a very short service time for propeller repair and replacement when malfunctions are minor (used here on the basis of field experience) is able to overcome adverse malfunction rates. A comparable situation arose in the first simulation when field maintenance was sufficiently flexible to overcome a high incidence of minor malfunctions.

Influence of Increased Propeller Repair Time

As a result of the findings of previous simulations, a more stringent propeller maintenance problem was posed by increasing the propeller repair time. This was done by increasing the frequency of occurrence of the 10- and 11-man-hour tasks (see Table 23, at the end of this section). It has been observed that maintenance repair times tend to cluster about certain discrete values representing the repair times for certain particular kinds of malfunctions. For existing turboprop systems, a 10- to 11-man-hour task might correspond to removal of the propeller to replace a vital component. In several subsequent cases, propeller repair times were increased still further to demonstrate their effect on operational capability.

An evaluation of this simulation run appears in Table 13, with a probability failure of only 0.008 per hour. At this level of reliability (equivalent to presently operating systems), the number of propeller malfunctions occurring is too few to affect the operational capability of the unit.

Influence of Combined Increases in Simulation Parameters

It is evident from the preceding discussion that reasonable variations in the values of any one of the simulation parameters are not sufficient to produce significant changes in operational capability from the standpoint of propeller reliability. It is therefore desirable to see the effect of increases in more than one of the parameters simulatneously.

The results of Simulation Runs 7, 8, and 9 are shown in Tables 14, 15, and 16, respectively. The conditions imposed on the operating unit in these cases include an average flying load of 1480 missions per month, mission durations of from 1 to 6 hours, probability of propeller malfunction 0.1, and three alternative schedules of propeller repair time, two previously introduced and the third of a more stringent nature.

In the operational situation for these three cases, unit capability is affected more by general maintenance limitations than by the propeller failure and maintenance process. However, this would not be unusual in real operations, where experience shows propeller systems seldom cause more than 10 per cent of the maintenance load. In Table 14, the frequency of propeller malfunctions is sufficient to cause some AOCP problems and infrequent queuing, but the propeller maintenance force is seldom fully occupied.

In Table 15, increasing the propeller maintenance task by increasing the frequency of occurrence of certain types of failures requiring a greater repair time places a substantial burden on the maintenance process. The unit is not meeting its operational requirement, but again this is mainly due to general maintenance. Comparing Table 14 with Table 15 shows that the increase in propeller maintenance does not influence unit capability significantly.

In Table 16, the propeller maintenance task has been made still more difficult. Now the full capacity of a two-shift operation is being utilized, but this is not sufficient to prevent aircraft delays in the propeller

TABLE 11. EVALUATION OF

Mission Duration
All other maintenance parameters are

	Operations					Propeller		
Day	Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	30	180	0	1	11	0	0
2	36	36	216	1	2	3	0	2
3	38	38	228	0	2	8	0	0
4	32	32	192	0	2	6	0	0
5	40	40	240	0	4	10	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	38	38	228	0	4	7	0	0
9	40	40	240	2	2	3	0	0
10	40	40	240	1	2	7	0	0
11	40	40	240	0	2	5	0	0
12	32	32	192	0	2	5	0	0
13	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0
15	38	38	228	1	3	8	0	0
16	36	36	216	0	2	4	0	0
17	32	32	192	0	2	5	0	0
18	32	32	192	1	1	5	0	0
19	32	32	192	0	2	3	0	0
20	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0
22	32	32	192	0	2	5	0	0
23	42	42	252	0	3	11	0	1
24	30	30	180	0	2	2	0	0
25	36	36	216	0	1	9	0	0
26	44	43	258	0	1	17	0	0
27	0	41	6	0	0	0	4	0
28	0	0	0	0	0	0	0	0
29	26	26	156	0	1	9	0	0
30	32	32	192	1	3	5	0	0

SIMULATION RUN 4

- 6 Hours
the same as for the basic case.

Maintenance		General Maintenance			
Man-Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	A/C in General Maintenance Queue	AOCP General	Man-Hours of General Maintenance Assigned	Number of General Maintenance Personnel Unassigned
57	30	0	0	587	0
18	30	0	7	691	71
10	30	0	8	578	59
18	30	1	10	376	51
38	27	0	11	844	78
0	21	0	8	0	19
0	30	5	3	0	94
23	30	8	1	825	100
13	28	0	6	860	63
32	28	0	10	899	47
11	23	0	12	817	34
11	25	0	14	567	25
0	27	0	15	0	37
0	30	5	10	0	96
35	30	10	5	855	100
6	23	0	6	679	60
34	30	0	5	668	54
11	27	0	6	538	52
12	30	0	6	602	63
0	30	0	6	0	79
0	30	2	4	0	93
17	30	4	2	733	100
24	30	0	5	739	63
2	29	0	11	688	60
58	29	0	14	553	48
60	21	0	15	840	47
0	13	3	14	0	25
0	30	6	9	0	94
27	30	10	5	730	100
33	30	1	4	510	73

TABLE 12. EVALUATION OF

Probability of Propeller
All other maintenance parameters are

Day	Operations: Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller: Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	30	76	0	1	16	0	0
2	32	32	69	0	1	18	0	2
3	30	30	73	0	1	18	0	0
4	24	24	46	0	1	14	0	0
5	34	34	70	0	0	14	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	30	30	88	0	1	17	0	0
9	38	38	88	0	2	19	0	0
10	48	48	116	2	4	27	0	0
11	44	44	95	0	2	23	0	0
12	28	28	58	0	3	16	0	0
13	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0
15	30	30	50	0	0	11	0	0
16	40	40	99	0	3	21	0	0
17	22	22	53	0	0	12	0	0
18	40	40	82	0	2	20	0	0
19	28	28	67	0	1	20	0	1
20	0	0	0	0	0	0	0	1
21	0	0	0	0	0	0	0	1
22	40	40	90	0	3	23	0	1
23	32	32	77	2	3	24	0	1
24	22	22	35	1	0	10	0	1
25	26	26	65	1	1	13	0	0
26	34	34	76	0	1	25	0	0
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	26	26	52	0	2	13	0	0
30	34	34	71	0	2	19	0	0

SIMULATION RUN 5

Malfunction - 0.1
the same as for the basic case.

Maintenance		General Maintenance			
Man-Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	A/C in General Maintenance Queue	AOCP General	Man-Hours of General Maintenance Assigned	Number of General Maintenance Personnel Unassigned
76	30	0	0	1026	0
62	29	0	6	861	59
53	30	0	5	615	66
33	30	0	9	445	74
40	30	0	9	503	89
0	30	0	12	0	92
0	30	5	7	0	100
59	30	9	3	748	100
64	30	0	6	346	79
88	30	0	10	780	88
83	30	0	14	1161	66
52	20	0	10	547	27
0	30	0	6	0	74
0	30	3	3	0	93
38	30	3	3	578	100
93	30	0	3	681	75
39	30	0	5	468	70
68	30	0	5	693	79
75	30	0	6	472	73
0	30	1	8	0	87
0	30	1	7	0	100
63	30	4	4	720	100
71	30	0	5	640	76
75	30	0	7	497	81
92	30	0	7	586	90
64	27	0	6	698	86
0	30	0	7	0	73
0	30	1	6	0	100
39	30	6	1	639	100
53	30	0	2	702	83

TABLE 13. EVALUATION OF

Propeller Repair Time -
All other maintenance parameters

Day	Operations					Propeller Maintenance		
	Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	30	69	0	2	3	0	0
2	36	36	76	0	2	1	0	3
3	44	44	100	0	1	4	0	1
4	38	38	85	0	2	1	0	1
5	52	52	126	0	1	2	0	1
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	36	36	85	0	1	3	0	0
9	40	40	105	0	3	3	0	0
10	30	30	77	0	1	1	0	0
11	34	34	78	0	1	2	0	0
12	38	38	86	0	3	2	0	0
13	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0
15	44	44	97	1	4	3	0	0
16	26	26	61	0	1	3	0	0
17	34	34	81	0	1	0	0	0
18	34	34	77	0	1	0	0	0
19	32	32	68	0	0	2	0	0
20	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0
22	42	42	96	0	1	1	0	0
23	34	34	91	0	2	1	0	0
24	32	32	71	0	2	3	0	0
25	28	28	73	0	1	3	0	0
26	44	44	126	0	2	4	0	0
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	28	28	71	0	2	4	0	0
30	40	40	83	0	1	2	0	0

OF SIMULATION RUN 6

Distribution II
are the same as for the basic case.

Man-Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	General Maintenance: A/C in General Maintenance Queue	General Maintenance: AOCP General	General Maintenance: Man-Hours of General Maintenance Assigned	General Maintenance: Number of General Maintenance Personnel Unassigned
61	30	0	0	828	0
12	27	0	9	761	66
28	30	0	8	551	74
10	30	0	14	1181	78
32	30	0	12	977	43
0	30	1	10	0	16
0	30	1	9	0	85
38	30	7	3	783	100
23	27	0	6	552	77
12	30	0	14	616	77
21	30	0	10	689	79
2	30	0	10	775	71
0	30	0	7	0	77
0	30	5	2	0	100
6	30	6	1	1009	100
25	30	0	7	566	73
0	30	0	9	560	79
0	30	0	15	641	82
2	30	0	16	931	72
0	30	0	10	0	54
0	30	5	5	0	96
12	30	7	3	876	100
0	30	0	4	533	71
13	29	0	9	566	83
14	30	0	9	564	75
26	30	0	10	670	72
0	30	0	10	0	72
0	30	3	7	0	98
24	30	4	6	591	100
3	30	0	3	587	87

TABLE 14. EVALUATION

Average Number of Missions per
Probability of
All other maintenance parameters

	Operations					Propeller Maintenance		
Day	Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	30	82	0	1	15	0	0
2	71	54	115	1	0	32	0	2
3	67	35	90	1	2	24	0	0
4	65	34	79	0	3	18	0	1
5	63	41	86	0	2	19	0	1
6	0	22	52	0	1	13	0	1
7	0	0	0	0	0	0	0	1
8	77	57	132	0	1	32	0	1
9	56	49	115	0	2	26	0	1
10	75	36	76	0	3	14	0	3
11	59	39	98	0	1	19	0	0
12	63	42	100	0	3	24	0	0
13	0	21	57	0	0	14	0	0
14	0	0	0	0	0	0	0	0
15	59	49	105	0	0	23	0	0
16	77	48	95	0	1	23	0	1
17	65	44	104	0	0	24	0	0
18	59	40	100	0	1	16	0	0
19	50	43	88	0	1	20	0	0
20	0	7	13	0	1	6	0	0
21	0	0	0	0	0	0	1	0
22	67	61	149	0	4	39	1	0
23	69	43	112	0	2	20	2	0
24	59	46	93	1	0	20	0	0
25	75	44	109	1	2	24	0	0
26	56	44	95	2	1	29	0	0
27	0	12	29	1	2	7	0	0
28	0	0	0	0	0	0	0	0
29	79	73	164	0	3	48	0	0
30	61	47	98	0	1	27	0	0

OF SIMULATION RUN 7

Month for Aircraft - 1480
Propeller Failure - 0.1
are the same as for the basic case.

		General Maintenance			
Man-Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	A/C in General Maintenance Queue	AOCP General	Man-Hours of General Maintenance Assigned	Number of General Maintenance Personnel Unassigned
76	30	0	0	796	0
105	30	0	13	1027	66
80	9	0	16	742	27
56	9	0	16	783	26
91	17	0	15	912	37
0	12	0	10	0	14
0	30	29	4	0	92
103	30	30	3	1759	100
86	20	0	5	990	3
28	17	0	10	623	24
118	19	1	12	761	32
69	16	0	11	812	17
0	16	0	12	0	43
0	30	24	10	0	97
159	30	26	8	1492	100
61	12	0	10	1083	4
87	19	0	9	809	12
48	15	1	13	697	17
65	16	0	15	732	27
0	11	0	17	0	32
0	30	10	15	0	93
157	30	16	9	1336	100
68	9	2	12	954	20
63	15	0	10	939	14
63	17	0	12	889	14
108	11	0	11	739	15
0	10	0	11	0	30
0	30	18	8	0	100
171	30	22	4	1456	100
97	0	0	9	832	18

TABLE 15. EVALUATION

Average Number of
Probability of
Repair Time -
All other maintenance parameters

	Operations					Propeller Maintenance		
Day	Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	30	80	0	5	18	0	0
2	73	0	141	1	2	32	0	2
3	67	36	74	2	0	17	0	1
4	77	44	84	0	1	26	0	1
5	67	40	92	0	1	22	0	2
6	0	23	49	0	1	13	0	1
7	0	3	3	0	0	0	0	1
8	69	57	138	0	3	27	0	1
9	65	47	110	1	3	25	0	1
10	65	42	95	2	3	23	0	0
11	69	33	87	0	0	19	0	1
12	75	32	85	0	0	27	0	0
13	0	33	80	0	3	19	1	0
14	0	5	0	0	0	0	1	0
15	65	58	129	0	2	31	1	0
16	75	45	105	0	1	24	0	0
17	56	41	93	0	2	27	0	0
18	65	41	90	0	5	21	1	0
19	50	46	105	0	0	27	0	0
20	0	4	7	0	0	3	0	0
21	0	0	0	0	0	0	0	0
22	69	68	159	0	3	44	0	0
23	73	38	118	0	2	24	0	0
24	71	36	74	0	4	24	1	0
25	50	35	79	0	2	18	0	0
26	59	32	70	1	1	18	0	0
27	0	27	58	0	2	13	0	0
28	0	0	0	0	0	0	0	0
29	53	46	102	0	1	23	0	0
30	71	52	111	1	4	30	0	0

OF SIMULATION RUN 8

Missions per Month - 1480
Propeller Failure - 0.1
Distribution II
are the same as for the basic case.

Man-Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	General Maintenance: A/C in General Maintenance Queue	General Maintenance: AOCP General	General Maintenance: Man-Hours of General Maintenance Assigned	General Maintenance: Number of General Maintenance Personnel Unassigned
151	30	0	0	571	0
221	30	0	8	1261	82
146	8	0	11	641	17
123	13	0	14	839	32
144	11	0	14	822	23
0	2	0	16	0	23
0	30	34	6	0	90
281	30	40	3	1782	100
156	2	0	7	1015	8
112	0	0	8	801	16
129	20	0	13	830	26
202	10	0	9	638	23
0	5	1	5	0	15
0	30	30	6	0	100
362	30	31	5	1474	100
128	5	0	9	898	11
169	12	0	8	639	14
137	2	1	10	838	28
229	8	0	10	859	30
0	4	0	9	0	9
0	30	8	5	0	98
272	30	13	0	1592	100
148	0	0	6	556	18
149	2	0	10	882	27
95	5	0	9	870	26
123	14	0	12	730	31
0	12	0	9	0	23
0	30	32	6	0	96
285	30	35	3	1877	100
164	6	0	6	970	6

TABLE 16. EVALUATION OF

Average Number of Missions
Probability of Propeller
Repair Time -
All other maintenance parameters

	Operations					Propeller		
Day	Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	30	66	0	0	17	0	0
2	73	60	137	1	3	38	0	2
3	63	32	73	1	1	19	1	0
4	65	25	64	0	4	15	1	0
5	67	32	72	0	0	21	0	0
6	0	18	62	0	1	13	0	0
7	0	1	2	0	0	0	3	0
8	63	29	64	0	2	19	3	0
9	71	32	64	0	2	16	1	0
10	59	39	75	0	2	24	0	0
11	56	41	117	0	1	30	1	0
12	65	24	49	0	3	13	1	1
13	0	23	41	0	2	9	1	1
14	0	0	0	0	0	0	3	1
15	56	40	92	0	2	22	2	1
16	71	39	81	0	1	22	0	0
17	71	36	69	0	4	18	0	1
18	67	40	86	0	1	19	2	0
19	65	38	80	0	1	21	0	0
20	0	18	41	0	1	12	0	1
21	0	3	11	0	0	1	0	0
22	56	38	88	0	2	22	0	0
23	56	37	86	0	1	20	0	0
24	75	41	90	0	1	20	3	0
25	73	39	78	0	3	22	1	0
26	71	39	101	0	3	19	1	0
27	0	24	59	1	1	16	2	0
28	0	0	0	0	0	0	2	0
29	15	40	94	0	5	24	1	0
30	77	32	97	1	1	22	0	0

SIMULATION RUN 9

per Month for Aircraft - 1480
Failure - 0.1
Distribution in III
are the same as for the basic case

Maintenance		General Maintenance			
Man Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	A/C in General Maintenance Queue	AOCP General	Man Hours of General Maintenance Assigned	Number of General Maintenance Personnel Unassigned
588	30	0	0	767	0
654	2	0	5	1276	82
454	0	0	6	729	27
229	0	0	9	557	40
493	2	0	14	693	47
0	0	0	11	0	32
0	30	18	12	0	96
1015	30	23	8	1013	100
261	0	1	7	843	39
313	0	0	4	561	32
620	0	0	9	654	43
364	0	0	13	571	44
0	0	0	11	0	44
0	27	28	9	0	97
624	27	32	4	1304	100
596	0	0	5	568	31
330	1	0	8	608	25
408	0	0	9	671	38
572	1	0	12	830	27
0	0	0	13	0	39
0	30	26	6	0	89
692	30	29	6	1090	100
477	0	0	7	823	24
568	0	1	8	600	30
391	1	2	9	737	34
389	0	0	10	817	34
0	0	0	12	0	24
0	30	25	13	0	99
616	30	32	6	1218	100
422	0	0	7	970	32

TABLE 17. EVALUATION OF

Average Number of Missions
Missions Duration –
Probability of Propeller
Repair Time –
All other maintenance parameters are

	Operations					Propeller		
Day	Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	30	180	0	2	27	0	0
2	73	39	234	0	0	34	0	2
3	50	36	216	0	2	31	3	0
4	69	22	132	0	1	21	5	3
5	79	26	156	0	1	23	0	1
6	0	14	84	0	0	10	1	1
7	0	1	6	0	0	1	1	0
8	69	28	168	0	2	25	0	0
9	81	24	144	0	2	22	2	0
10	69	26	156	0	2	25	0	1
11	67	22	132	0	1	21	6	1
12	65	19	114	1	0	18	5	0
13	0	13	78	0	0	12	2	0
14	0	0	0	0	0	0	5	0
15	61	22	132	0	1	19	3	0
16	71	18	108	0	1	16	1	1
17	73	24	144	0	0	23	4	0
18	53	22	132	0	0	16	2	0
19	73	30	180	0	1	27	4	0
20	0	13	78	0	1	13	3	0
21	0	1	6	0	0	1	4	0
22	56	22	132	0	0	21	2	0
23	71	29	174	0	1	26	3	0
24	69	30	180	0	1	25	2	0
25	67	26	156	0	1	26	4	0
26	69	21	126	0	2	20	6	1
27	0	11	66	0	1	11	0	1
28	0	1	6	0	0	0	1	2
29	73	29	174	0	2	27	1	1
30	75	25	150	0	0	22	2	1

SIMULATION RUN 10

per Month - 1480
6 Hours (Fixed)
Failure - 0.1
Distribution III
the same as for the basic case.

Maintenance		General Maintenance			
Man-Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	A/C in General Maintenance Queue	AOCP General	Man-Hours of General Maintenance Assigned	Number of General Maintenance Personnel Unassigned
605	30	0	0	834	0
656	0	0	7	944	57
534	0	4	2	552	15
372	0	7	6	570	27
475	0	0	7	463	39
0	0	1	7	0	42
0	30	15	5	0	93
1690	30	19	2	827	100
502	0	1	5	473	41
488	0	0	4	546	56
376	0	5	6	310	44
343	1	0	8	441	62
0	0	1	6	0	51
0	30	13	7	0	94
609	30	16	4	924	100
348	0	2	3	405	33
340	0	1	6	499	42
286	0	0	5	356	52
558	0	3	7	521	58
0	0	1	8	0	31
0	30	12	9	768	96
721	30	0	5	0	43
369	0	16	6	578	100
587	0	1	5	532	37
554	0	2	8	492	41
477	0	3	11	229	53
0	1	0	11	0	57
0	30	14	9	0	99
744	30	21	3	886	100
564	0	3	2	336	40

TABLE 18. EVALUATION OF

Mission Duration -
Repair Time -
Propeller Maintenance

Day	Operations					Propeller		
	Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	30	180	1	0	2	0	0
2	36	36	216	0	1	7	0	2
3	30	30	180	0	2	5	0	0
4	32	32	192	0	1	11	0	0
5	34	34	204	0	1	5	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	36	36	216	1	0	9	0	0
9	38	38	228	0	1	7	0	0
10	26	26	156	0	2	6	0	0
11	32	32	192	0	3	10	0	0
12	24	24	144	0	3	8	0	0
13	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0
15	42	42	252	0	2	6	0	0
16	36	36	216	0	1	5	0	0
17	44	44	264	0	3	6	0	0
18	34	34	204	0	0	4	0	0
19	40	40	240	0	1	4	0	0
20	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0
22	20	20	120	0	1	4	0	0
23	22	22	132	0	2	5	0	0
24	28	28	168	0	1	3	0	0
25	40	40	240	1	3	6	0	0
26	36	36	216	0	1	5	0	0
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	32	32	192	0	3	3	0	0
30	44	44	264	0	0	12	0	0

SIMULATION RUN 11

6 Hours (Fixed)
Distribution III
Personnel – 15

Maintenance		General Maintenance			
Man-Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	A/C in General Maintenance Queue	AOCP General	Man-Hours of General Maintenance Assigned	Number of General Maintenance Personnel Unassigned
62	30	0	0	838	0
255	12	0	7	698	68
125	1	0	8	663	63
335	4	0	9	502	54
107	0	0	6	727	61
0	0	0	8	0	51
0	15	2	6	0	96
328	15	5	3	615	100
266	1	0	4	520	86
52	3	0	8	514	74
237	3	0	6	570	73
84	1	0	8	519	69
0	5	0	7	0	70
0	15	2	5	0	100
100	15	2	5	687	100
115	8	0	9	1036	70
184	5	0	5	728	51
64	0	0	11	463	39
168	9	0	10	758	52
0	1	0	12	0	39
0	15	1	11	0	100
112	15	6	6	426	100
118	7	0	6	436	76
56	3	0	3	485	83
54	11	0	5	717	74
105	12	0	9	686	58
0	4	0	8	0	53
0	15	2	6	0	94
139	15	6	2	496	100
243	2	0	6	641	78

TABLE 19. EVALUATION OF

Mission Duration –
Probability of Propeller
Repair Time –
Propeller Maintenance

	Operations					Propeller		
Day	Missions Generated	Missions Flown	Total Flying Hours	Propeller Aborts	Other Aborts	Propeller Malfunctions	A/C in Propeller Maintenance Queue	AOCP Propeller
1	--	30	180	1	3	26	0	0
2	30	29	174	1	1	27	2	5
3	36	17	102	0	2	16	2	3
4	28	15	90	0	1	15	2	2
5	36	15	90	0	1	13	3	1
6	1	7	42	0	1	7	4	2
7	0	0	0	0	0	0	4	0
8	36	13	78	0	1	12	4	0
9	34	16	96	0	0	15	1	1
10	32	14	84	0	1	13	6	0
11	42	14	84	0	1	13	4	0
12	34	20	120	0	1	18	4	0
13	0	10	60	0	0	10	1	0
14	0	0	0	0	0	0	3	0
15	32	13	78	0	0	12	2	0
16	40	17	102	0	1	17	4	0
17	40	12	72	0	0	10	5	0
18	22	14	84	0	0	11	7	0
19	38	10	60	0	1	10	7	0
20	0	6	36	0	0	5	4	0
21	0	0	0	0	0	0	3	0
22	40	13	78	0	1	11	2	0
23	34	17	102	0	0	15	2	1
24	30	11	66	0	0	11	1	1
25	36	13	78	0	0	12	2	0
26	38	13	77	0	0	10	3	0
27	0	10	60	1	0	9	1	0
28	0	0	0	0	0	0	1	0
29	38	12	72	0	0	12	1	0
30	32	12	72	0	0	12	2	0

SIMULATION RUN 12

6 Hours (Fixed)
Failure - 0.1
Distribution III
Personnel - 15

Maintenance		General Maintenance			
Man-Hours of Propeller Maintenance Assigned	Number of Propeller Maintenance Personnel Unassigned	A/C in General Maintenance Queue	AOCP General	Man-Hours of General Maintenance Assigned	Number of General Maintenance Personnel Unassigned
573	30	0	0	715	0
496	0	0	11	599	58
337	0	0	9	476	49
258	0	1	8	301	50
307	0	1	6	192	54
0	0	1	8	0	72
0	15	11	5	0	100
420	15	12	4	452	100
291	0	0	3	191	68
200	0	1	7	343	66
296	0	2	7	395	71
273	0	2	6	457	10
0	0	0	6	0	38
0	15	11	5	0	98
471	15	13	3	590	100
513	0	0	5	319	71
198	0	0	5	198	64
278	0	1	7	265	72
90	0	0	10	241	70
0	0	0	8	0	77
0	15	9	5	0	96
313	15	12	2	476	100
201	1	0	3	259	74
246	0	1	4	265	73
232	0	0	3	396	71
141	0	1	3	263	59
0	0	1	1	0	69
0	15	10	1	0	95
451	15	10	1	340	100
263	0	0	2	203	71

maintenance queue. The relative burden of propeller maintenance is high in comparison with the general maintenance. Propeller reliability in this case has a significant influence on unit operational capability.

In all three cases, the number of missions flown is indicative of the maximum capability of the unit under the specified conditions. In Simulation Runs 7 and 8 (Tables 14 and 15), the unit is able to accomplish approximately 1,030 missions. However, under the more stringent propeller repair requirement in Simulation Run 9 (Table 16), the unit is able to accomplish only 850 missions. Since the only parameter varied in these three cases is the propeller repair time requirement, the reduction in mission capability can be attributed directly to propeller reliability.

In the simulation shown in Table 17, the operational conditions are the same as those for Table 16 except the mission duration is now assumed to be 6 hours for all flights. The propeller maintenance activity in this case is considerably overstressed, even on the two-shift basis. The number of aircraft awaiting maintenance in the propeller queue indicates that this condition would induce a severe availability problem. The operational capability of the unit is now reduced to only 594 missions in the 30-day period.

The final simulation runs, Runs 11 and 12, are presented in Tables 18 and 19. These two cases compare the effects of low and high probability of propeller malfunction when the propeller maintenance force is only half its previous size, or 15 men. Other conditions include an average flying load of 740 missions per month, each of 6 hours' duration, and the most stringent case for propeller maintenance time.

In Table 18, the maintenance force is almost fully utilized in accomplishing the 736 missions actually flown. The operational capability of the unit is sufficient to accomplish all missions assigned. Reliability of the propeller is acceptable, but malfunctions that do occur are difficult to repair. In Table 19, when the failure rate is increased to a probability of 0.1 per hour, the propeller maintenance force is not sufficient. Aircraft waiting in the propeller maintenance queue are largely responsible for the decline in unit capability. The unit is able to accomplish only 343 missions. This situation might be viewed as a detached unit operation, and indicates the effect of propeller reliability under conditions of limited maintenance.

Description of the Digital Computer Program Used to Determine the Disposition of Aircraft During a Period of Operations

The program for digital computation was prepared for the purpose of analyzing the operations of a unit with a complement of 50 turboprop aircraft for a period of, say, 30 days when the change of aircraft disposition is considered on an hourly basis. The computer program is best described by the flow of information through the computer. Flow chart illustrations shown in Figures 41 through 46 present, in simplified form, the sequence of events that has been converted to machine instructions. These flow charts will be described as they are introduced in the discussion.

The flow of information was developed from a mathematical representation of the several events of the operational sequence. The model is based upon the underlying assumptions previously discussed. Most of the computational routines consist of a simple accounting of time, aircraft, men, and missions. When random events are considered, it is necessary to employ Monte Carlo techniques. Random numbers are generated and are used as the arguments for entries to tables containing the appropriate functional values stored in the computer memory. This technique is employed to determine functional values for the following:

(1) The mission requirements generated during each day of operations (Table 6)

(2) The duration of the mission to be flown (Table 6)

(3) Whether or not a mission is aborted

(4) Whether there is a propeller failure

(5) Frequency and duration of AOCP (general and/or propeller, Table 7)

(6) The man-hours and the number of men required for general and propeller maintenance (Tables 8 and 9).

The following conventions are used in the discussion of the program and in the flow charts:

a/c Aircraft

AOCP Aircraft out-of-commission, parts

◇ A point in the program where alternative courses of action can be followed

○ Interconnecting, entry, or exit points in the program.

Representation of the Simulation by the Computer Program

The following discussion presents the essential details of the computer program used to simulate the operational sequence of events.

Starting the Program

At the beginning of each hour and/or day, and at the initial start of the program, a group of calculations are performed before referring to any particular aircraft. In Figure 41, the first step in the computations is to take the number of missions aborted during the preceding hour and add these to the number of unassigned missions remaining for the particular day. Note that these missions are not identified by length at this point. A certain number of missions are preassigned as an initial condition for the computation. At 0800 of each succeeding day, a Monte Carlo method is used to determine the number of new missions generated for the day's operations.

To obtain the daily mission requirements, the random number argument is used to enter a stored table with functional values obtained as discussed below. The number of missions obtained is based on the assumption that a certain number of missions are flown each month. An average value for normal operation is taken to be 740 missions. A value of 1480 missions can be used to study the effect of a doubled mission requirement. One method of realizing this number of missions is to derive a probability of flight of an aircraft for each hour of operations and then perform a separate Monte Carlo each time an aircraft is found available. However, this laborious process can be avoided, since the individual probabilities are low and repetition of the process a large number of times would be expected for an operational period of 30 days. Therefore, it is possible to use Poisson's exponential binomial limit in the form:

$$P\ (\text{flights}) = \frac{e^{-Np}(Np)^k}{k!},$$

wherein the terms have the following definitions:

- N Number of aircraft (50)
- p Probability of flight of each aircraft during a 24-hour period
- k Number of missions generated for 24 hours of operations.

For the 740- and 1480-mission cases, Np is found to be 33 and 66, respectively. In the first case, the probability of occurrence is not greater than or equal to 0.0001 until k equals 14. Only four significant digits are used in the Monte Carlo process employed, and 14 is the least number of missions generated for a day's operations. Also, when k exceeds 56, the probability again becomes less than 0.0001, and 56 is the maximum number of missions generated. The corresponding numbers for the 1480-mission case are 40 and 93. The relative frequencies used are found in published tables.(1)

(1) Molina, E. C., Poisson's Exponential Binomial Limit, D. Van Nostrand Company, Inc., New York (1942).

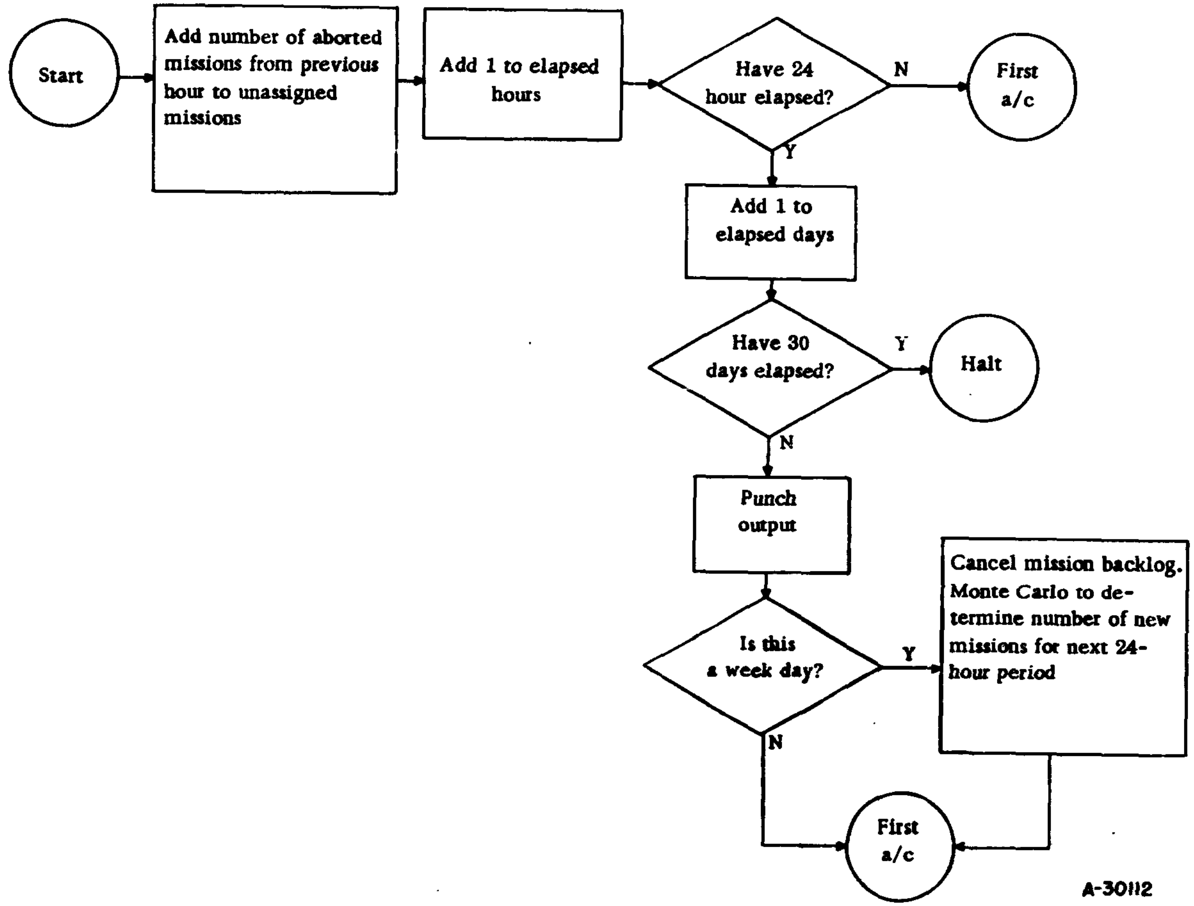

FIGURE 41. START OF PROGRAM FOR ANALYSIS OF AIRCRAFT DISPOSITION

New missions are generated on the first 5 days of the week only. The mission backlog is cancelled at 0800 of each week day to prevent an unrealistic accumulation of backlog missions. On the 6th and 7th days (week ends), the mission backlog remaining from the 5th day's (Friday) operations is flown. This represents a reduced operational effort for Saturday and Sunday. When the 30-day period of interest has elapsed, the simulation is halted.

The output (or punch) instructions are also found in this block. The data output at 0800 of each day is as follows:

(1) Day

(2) Number of new missions generated for day's operations

(3) Number of missions generated at 0800 of preceding day but not flown by 0800 of current day

(4) Number of missions aborted during preceding 24 hours because of propeller failures

(5) Number of missions aborted during preceding 24 hours because of nonpropeller failures

(6) Number of propeller failures (in flight) during preceding 24 hours

(7) Man-hours of propeller maintenance work in progress but not completed

(8) Man-hours of general maintenance work in progress but not completed

(9) Number of aircraft in general maintenance queue

(10) Number of aircraft in propeller maintenance queue

(11) Number of aircraft AOCP (propeller)

(12) Number of aircraft AOCP (general maintenance)

(13) Man-hours of general maintenance work assigned during preceding 24 hours

(14) Man-hours of propeller maintenance work assigned during preceding 24 hours

(15) Total number of aircraft hours available, i.e., sum of aircraft available for each of preceding 24 hours

(16) Total number of flight hours during preceding 24 hours

(17) Unassigned general maintenance men

(18) Unassigned propeller maintenance men

(19) Total flight time of each aircraft.

The output is designed to make available the most significant aspects of a day's operations.

Computational methods are not shown in the flow charts, since this unnecessarily complicates the presentation and only some simple bookkeeping is involved. A tally is kept of the number of missions aborted during the preceding 24 hours, and, after these data are punched, the computer memory location in which the tally is stored is reset to zero to start a new count for the next 24 hours of operations.

Processing of Individual Aircraft (A/C) Records

The entry FIRST A/C refers to the first aircraft record, this being the exit from the "START" flow chart (Figure 38). The first aircraft record is examined to determine the category of the aircraft and, where applicable, the time the aircraft must remain in that category. The category and the time to remain therein determine the disposition of the aircraft during the next hour. The flow charts in Figures 41 through 46 are concerned with the processing of individual aircraft records. After the records of all 50 aircraft have been processed, the computer program returns to the "START" block to compute the events of the next hour. The primary categories in which aircraft are placed are as follows:

(1) Available for flying a mission

(2) In flight

(3) In flight with a propeller failure

(4) AOCP for parts required in general maintenance

(5) AOCP for parts required in general maintenance, and with a propeller also in maintenance channels

(6) In the general maintenance queue

(7) In the general maintenance queue, and with a propeller also in maintenance channels

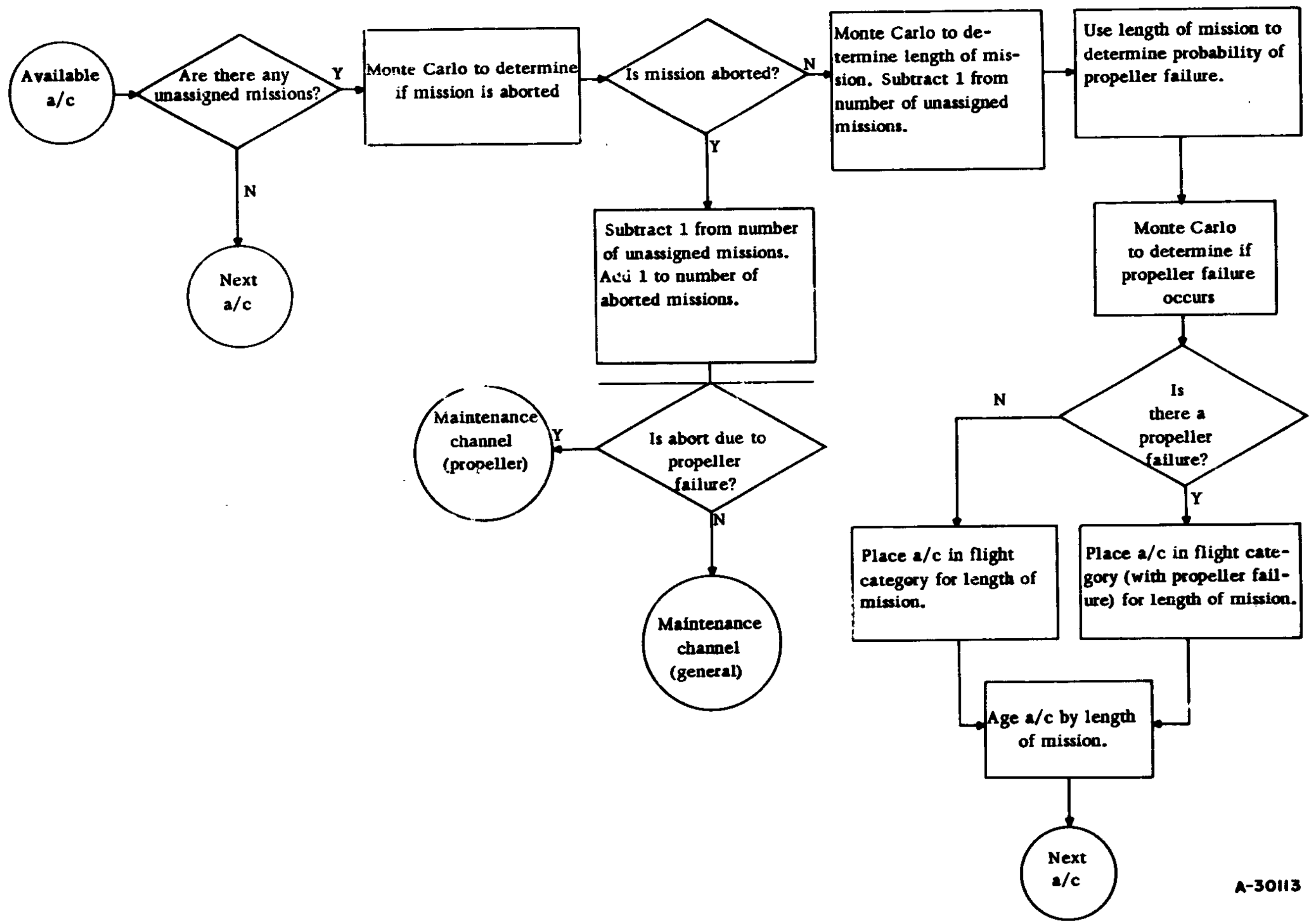

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FIGURE 42. ASSIGNMENT OF AVAILABLE AIRCRAFT

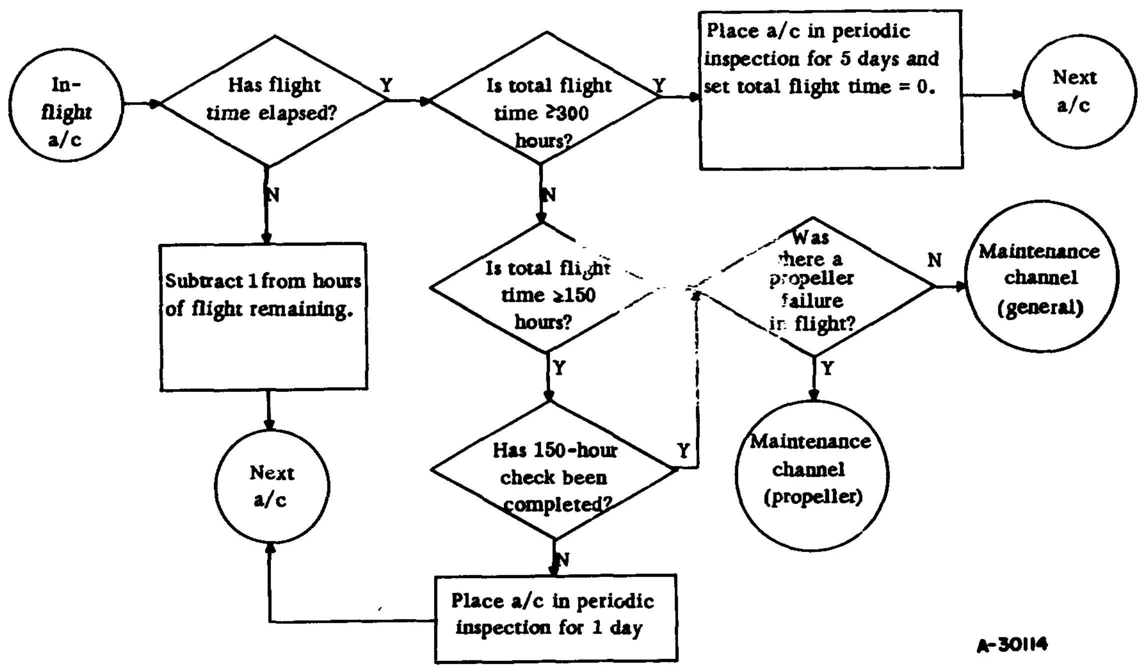
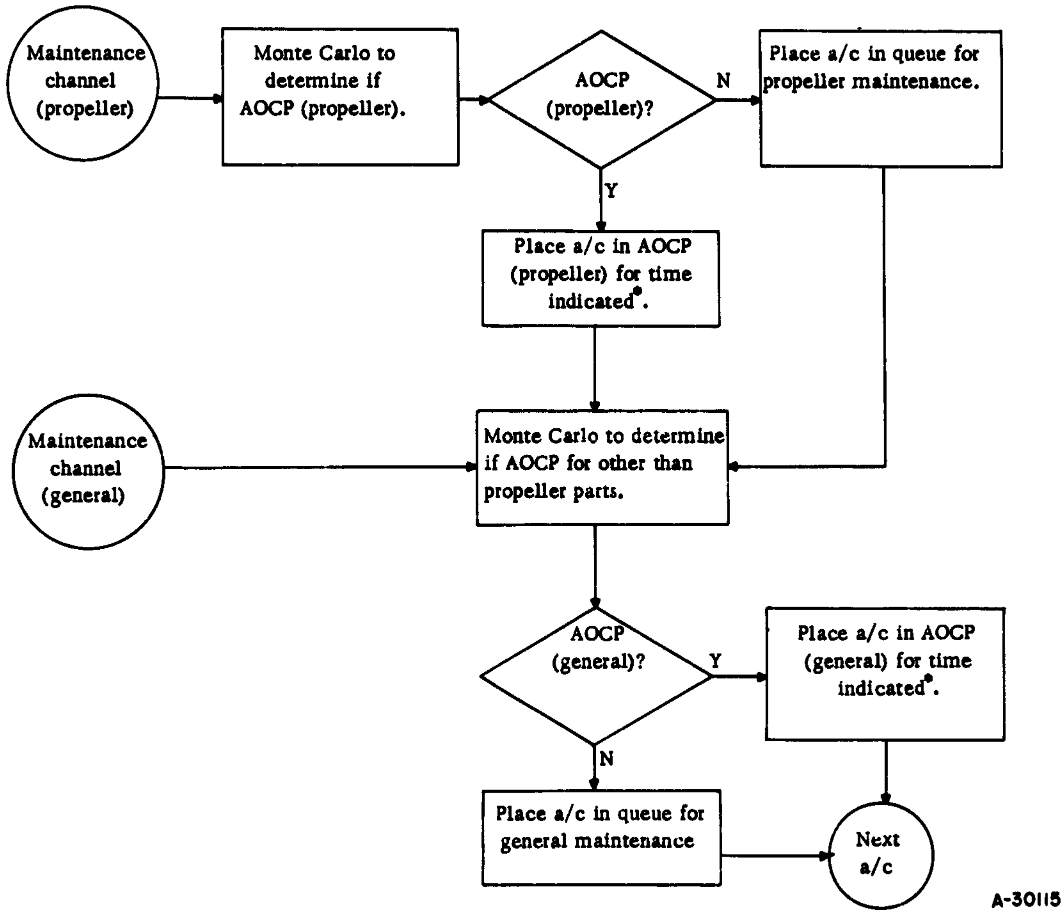
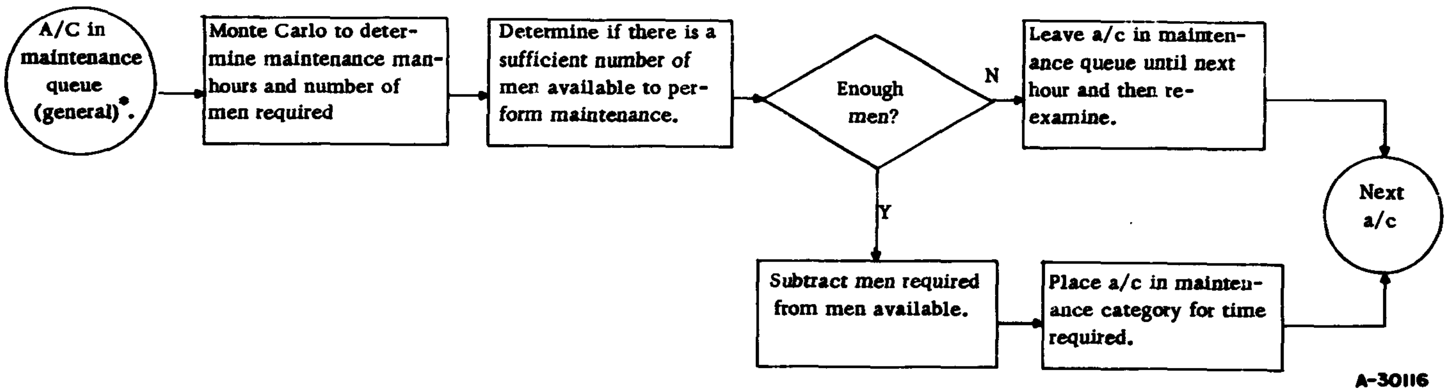


FIGURE 43. ASSIGNMENT OF AIRCRAFT RETURNING FROM FLIGHT



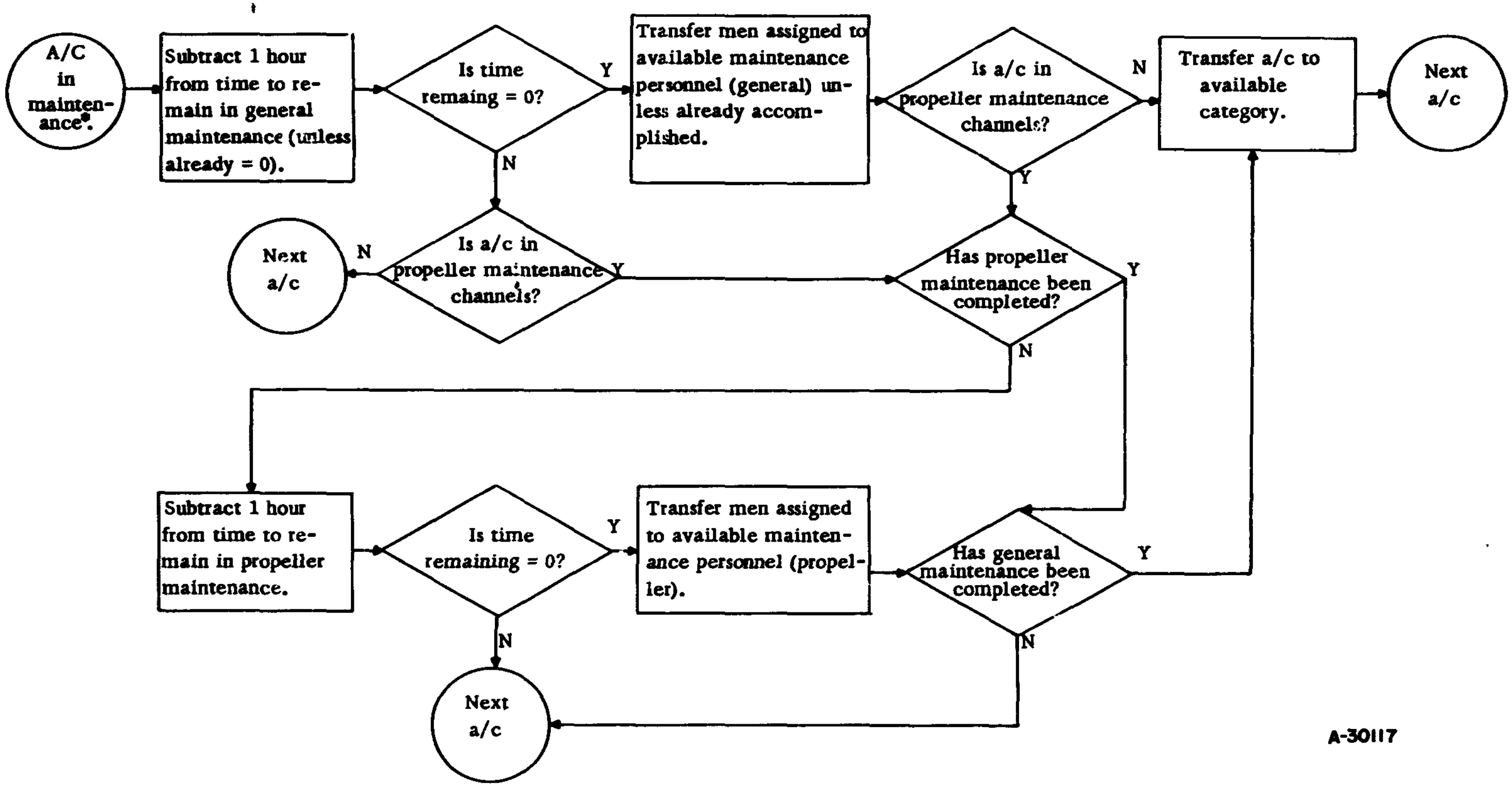
* When time in AOCP is completed, aircraft is transferred to appropriate queue.

FIGURE 44. START OF MAINTENANCE CHANNELS (PROPELLER AND GENERAL)



* If it is a week-end day, new maintenance assignments are not made. Aircraft in the propeller maintenance queue are considered in an identical manner.

FIGURE 45. MAINTENANCE QUEUE (GENERAL AND PROPELLER)



* This category is bypassed if time of day is past working hours.

FIGURE 46. AIRCRAFT IN MAINTENANCE (GENERAL AND PROPELLER)

(8) In general maintenance

(9) In general maintenance and with a propeller in maintenance channels also

(10) General maintenance completed, but with a propeller remaining in maintenance channels

(11) Undergoing periodic inspection (150- or 300-hour check).

These categories are inclusive. Each aircraft must always be present in one of the above categories. However, if propeller maintenance is being performed, an aircraft can also be present in one of the following additional (secondary) categories:

(1) AOCP for parts required in propeller maintenance

(2) In the propeller maintenance queue

(3) In propeller maintenance

(4) Propeller maintenance completed, but still in general maintenance channels.

Individual aircraft records also carry information on the time to remain in the primary and secondary categories, the number of men assigned for both general and propeller maintenance, the total flight hours, and whether or not the 150-hour check has been completed.

Processing of Available Aircraft. Available aircraft records are processed according to Figure 39. However, if there is no mission backlog, the aircraft record is not processed, and the computation proceeds to the next aircraft record. It should be noted here that at any point in the flow charts where NEXT A/C is indicated, the processing of a particular aircraft record is complete for that hour. The next aircraft record is then considered, and this process repeated for each aircraft in each hour.

When missions are to be flown, the Monte Carlo technique is used to determine whether the mission is aborted. This is accomplished by generating a four-digit random number, ascertaining whether this number is less than a given value, say, 0500, and, if so, the mission is regarded as aborted. In this example, abort rate would be equivalent to 5 per cent. The random number is also compared with a smaller number, say 0050, and, if less than this, the abort is considered due to a propeller malfunction. The implication here is that 4.5 per cent of the missions are aborted because of general malfunctions and the other 0.5 per cent are propeller aborts. Note that, for simplification in record keeping, the length of the mission has not yet been determined. An aborted mission is not identified as to length, and does not accrue flying time.

The Monte Carlo technique is next applied to determine the length of the mission. The relative frequencies of various mission lengths found in Table 20 are based on actual flight records. The generated length of the mission is used to establish the probability of propeller failure. Although four propellers are under consideration, it is assumed that only one failure will occur per flight. This is not considered too severe an assumption, since multiple failures are relatively infrequent in actual operations, and they should therefore not greatly disturb the over-all propeller maintenance activities generated by single failures. To obtain the probability of a propeller failure for the aircraft in flight, the individual propeller failure probability is multiplied by 4, so that all propellers are considered. The probability of failure is assumed to be exponential and is determined by the expression:

$$1 - e^{-at} ,$$

wherein a is the hazard rate for four propellers per hour of flight, and t is the duration of the flight. A random selection is made to see whether the failure is realized. The two values of a used in this study were (4 x 0.008), or 0.032, and (4 x 0.1), or 0.4. The first value reflects specific operational experience with turboprop aircraft. After determining whether there is a propeller failure, the aircraft being considered is placed in in-flight status for the duration of the flight and is aged accordingly.

TABLE 20. RELATIVE FREQUENCIES OF VARIOUS MISSION LENGTHS

Mission Length, hours	Relative Frequency
1	0.3652
2	0.2767
3	0.1586
4	0.1197
5	0.0496
6	0.0301

Processing of In-Flight Aircraft. When a flight is completed, the aircraft record is processed as indicated in Figure 40. Aircraft age is checked to see whether either the 150-hour or the 300-hour periodic inspection is required. If the 300-hour check is due, the aircraft is placed in the periodic inspection category for a period of 5 days and then returned to the available list with its flight time reset to zero. For the 150-hour check, the same is done, but for 1 day only, and the total flight time is not reset.

If inspection is not due, the aircraft record is processed through the MAINTENANCE CHANNEL-GENERAL or both the MAINTENANCE CHANNEL-GENERAL AND MAINTENANCE CHANNEL-PROPELLER (Figure 41), depending on whether or not a propeller failure has occurred.

It should be noted here that all aircraft returning from a flight require some general maintenance. If a propeller failure has occurred, or if the mission was aborted because of a propeller malfunction, the aircraft is required to enter both maintenance channels, and it is not released to available status until both types of maintenance have been completed. The processing through both channels is simultaneous but not parallel. One type of maintenance may be completed before the other, e.g., an aircraft may be AOCP (propeller) while the general maintenance work is being performed.

Processing of Aircraft in Propeller and General Maintenance Channels

Upon entering propeller maintenance channels (Figure 41), a random-number comparison determines the occurrence and duration of an AOCP. The number of hours AOCP is based upon a Poisson distribution of time duration using discrete 12-hour intervals and a most probable value of 72 hours (Table 21). If AOCP time is zero, the aircraft is placed in the propeller maintenance queue until the next computational cycle at the next hour. The next step is to determine whether there is an AOCP in general maintenance. This is done on the same basis as for the propeller. Again, if AOCP is zero, the aircraft is placed in the general maintenance queue. Note in Figure 41 that aircraft without propeller malfunctions are processed only through the entry point MAINTENANCE CHANNEL (GENERAL).

Processing of Aircraft in the Maintenance Queues. Aircraft in the maintenance queues (general or propeller) are processed as shown in Figure 42. The number of maintenance man-hours required is selected randomly from a prescribed distribution. The maintenance man-hour data are based on empirical information obtained from aircraft maintenance records. Table 22 shows the relative frequency of maintenance man-hours for general maintenance. The number of men assigned to a specific maintenance job varies from one to four, and if more than one man is assigned, the number of hours the aircraft remains in the maintenance category is reduced accordingly. However, before the aircraft is placed in an active maintenance category, a test is made to see whether enough unassigned men are available to perform the job. If not, the aircraft remains in the maintenance queue until enough men become available. Although Figure 42 is concerned with general maintenance queues, the same procedure is followed in processing the propeller queue.

TABLE 21. RELATIVE FREQUENCIES OF HOURS OF AOCP

Hours AOCP	Relative Frequency	Hours AOCP	Relative Frequency
0	0.9002[a]	96	.0103
12	0.0015	108	.0069
24	0.0045	120	.0041
36	0.0089	132	.0023
48	0.0134	144	.0011
60	0.0161	156	.0005
72	0.0161	168	.0002
84	0.0138		

(a) This value is for a 10 per cent probability: if there were 30 per cent AOCP's, this value would be 0.7008, and other frequencies would be tripled.

TABLE 22. RELATIVE FREQUENCIES OF MAN-HOURS REQUIRED FOR GENERAL MAINTENANCE

Man-Hours Required	Relative Frequency	Man-Hours Required	Relative Frequency
1	.0081	16	.0356
2	.0360	18	.0291
3	.0397	20	.0449
4	.0478	25	.0473
5	.0478	30	.0437
6	.0652	35	.0433
7	.0664	45	.0469
8	.0563	55	.0324
9	.0579	65	.0162
10	.0797	75	.0146
12	.0793	85	.0073
14	.0445	95	.0101

Table 23 presents the relative frequencies of man-hour efforts required for propeller maintenance. Distribution I represents empirical data derived from aircraft maintenance records. Distribution II and III are assumed distributions representing higher frequencies of more difficult maintenance tasks. Distribution II considers an increased frequency of jobs requiring 10 to 11 man-hours of effort, whereas Distribution III describes more intensive maintenance effort, with tasks requiring up to 60 man-hours and the relative frequencies following an irregular pattern.

Processing of Aircraft in General and Propeller Maintenance. Figure 43 indicates the processing of aircraft in maintenance. When the time to remain in maintenance equals zero, the aircraft and the men assigned are returned to the available list. The complexities of this block are introduced in ascertaining whether both general and propeller maintenance are completed before transferring the aircraft to an available status. This category of aircraft is not processed if the hour of the day is beyond working hours. For the analyses performed to date, it has been assumed that two 8-hour shifts with an equal number of men assigned are being employed. A variation in the length of the working day can be accomplished by altering one constant in the program.

Processing of Aircraft in AOCP and Periodic Inspection Categories. The flow of computations for AOCP aircraft is not diagrammed, since this is handled on a relatively simple basis. Each hour the aircraft record is examined to see whether AOCP time has expired. If so, the aircraft is transferred to the appropriate maintenance queue. The same procedure is used for aircraft in a periodic inspection status, except that they are transferred to an available category.

Details of Program Operations

In running the program described, it was found that about 35 minutes were required to analyze a 30-day operational period on an hourly basis. The output was punched at 0800 of each day, since the hourly variations of aircraft disposition were not considered to be significant. Also, by not punching for each hour, the speed of computation was considerably improved, since excessive output retards the computational speed. Examination of hourly data would also be a tedious process, and it is therefore desirable to generate the minimum amount of data that will provide meaningful answers in the light of the desired investigation.

TABLE 23. RELATIVE FREQUENCIES OF MAN-HOURS REQUIRED FOR PROPELLER MAINTENANCE

Distribution I (Empirical)		Distribution II (Assumed)		Distribution III (Assumed)	
Man-Hours	Relative Frequency	Man-Hours	Relative Frequency	Man-Hours	Relative Frequency
1	0.2700	1	0.2000	1	0.2000
2	0.2700	2	0.2000	2	0.1000
3	0.1100	3	0.0167	3	0.0084
4	0.1100	4	0.0167	4	0.1056
5	0.0700	5	0.0167	8	0.0056
6	0.0300	6	0.0167	9	0.0083
7	0.0300	7	0.0167	12	0.0250
8	0.0300	8	0.0165	16	0.0139
9	0.0300	9	0.0167	18	0.0250
10	0.0100	10	0.2000	27	0.2167
11	0.0100	11	0.2000	32	0.0083
12	0.0100	12	0.0167	36	0.1167
13	0.0050	13	0.0167	48	0.1166
14	0.0050	14	0.0167	64	0.0499
15	0.0050	15	0.0167		
16	0.0050	16	0.0165		

The computations were performed with an IBM 650 Data Processing System. Additional indexing accumulators were utilized to reduce the number of instructions required and to increase the speed of calculation. The Symbolic Optimal Assembly Program[1] was used to optimize the location of instructions, this being helpful in reducing the time required for computation and in simplifying the coding.

(1) Poley, Stan, SOAP II Programmers' Reference Manual, Applied Science Division Publication, International Business Machines Corporation, 590 Madison Avenue, New York, New York.